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MENSURATION

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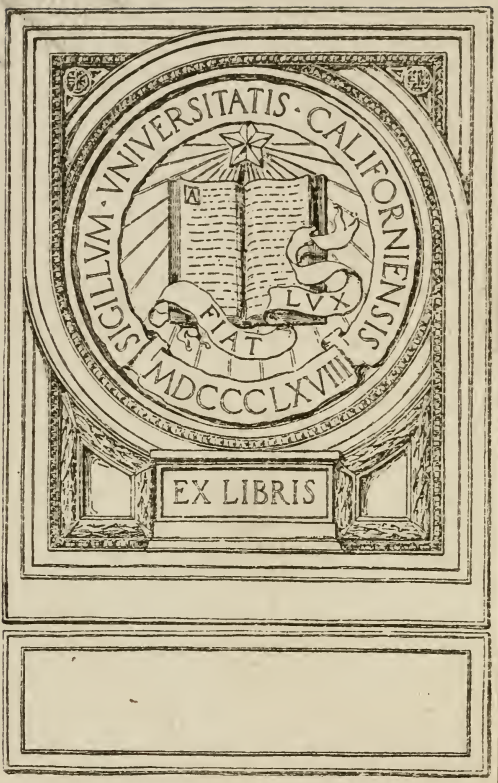
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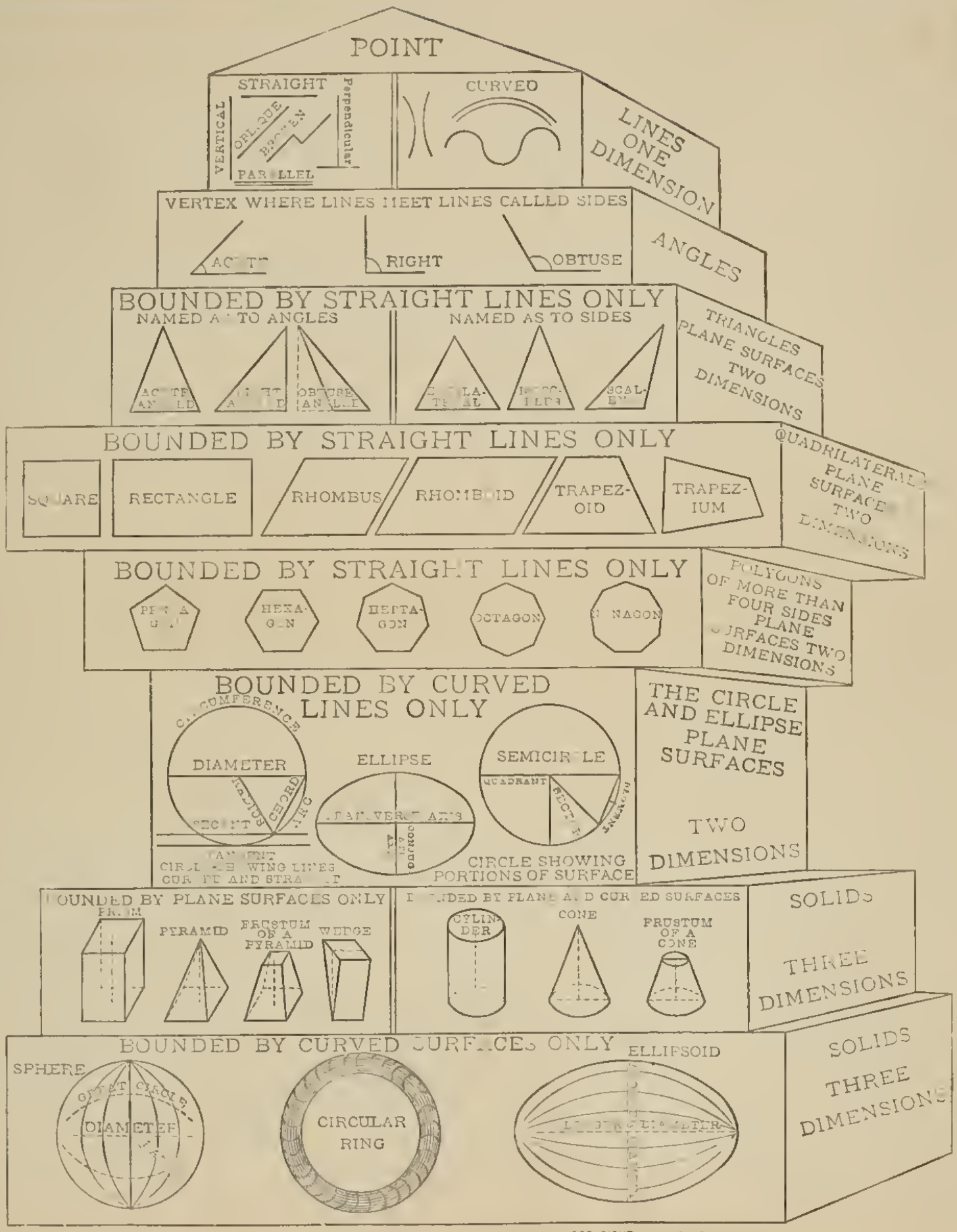

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FUNDAMENTALS OF MEASUREMENT.

MENSURATION

WITH SPECIAL APPLICATION OF THE PRISMOIDAL FORMULA

DESIGNED FOR SUPPLEMENTARY WORK IN
NORMAL SCHOOLS, HIGH SCHOOLS, ACADEMIES, AND ADVANCED
CLASSES IN GRAMMAR AND COMMON SCHOOLS

BY

S. W. FURST

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1911

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PREFACE

A NEW text-book at the close of the nineteenth century is no novelty. The author, therefore, without apology sends forth this little volume, hoping that his co-workers may find at least some of the pleasure and profit in using it that he has found in its preparation and practical application in the school-room.

The method of treating Mensuration, as here presented, has had a thorough class-room test—a safe criterion for any method. Both those pupils with a knowledge of Geometry and those without such knowledge have been able to solve problems with facility. Without this test with pupils of various ages and grades, the author would hesitate long before advising the setting aside of some of the standard rules used in text-books for years. Pupils who have become familiar with this method have invariably become enthused with the work. They enter examinations in this department confident of success, because they have become *masters of one* broad principle covering a great field of measurement. What was vague becomes clear; the concrete supplants the abstract. Concentration and simplicity are everywhere the central ideas.

A few words concerning the scope of this principle may not be amiss. As the title-page indicates, the special feature of this work is the application of the *prismoidal formula* to finding the volume of solids. This is used by civil engineers in making calculations for railway embankments, excavations, etc.; but, so far as we are aware, no common school text-book treats the

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subject at length. It applies to many other solids than those which fall strictly within the definition of a prismoid. Ellwood Morris, a civil engineer, made known this extended application about 1840:

"It applies to all solids having two parallel sides or faces, where these two faces, curved or plane, are united by surfaces upon which and through every point of which a straight line may be made to connect the two parallel faces; to prisms, wedges, pyramids, cones, etc., regular or irregular, right or oblique; to the sphere, hemisphere, and other spherical segments; to either section of a cylinder, where the cutting plane is passed through both parallel sides; to any section of a cone, where the cutting plane is passed through both apex and base; to frustums of prisms, pyramids, cones, etc." Volumes of frustums of pyramids and cones can be found by its use without the extraction of a root. It is thus seen that its scope is very great. For the derivation of this formula from the principles of Geometry, see Supplement.

Care has been taken to use only such rules and definitions as have been sanctioned by the best authorities. In order that comparisons may be readily made without reference to other books, and that the teacher may conveniently make selections of those which best suit his purpose, a number of these are given under each chapter.

The work abounds in interesting problems, many of which have been suggested by objects familiar to every boy and girl. Actual measurements were made for many of them. Both the grading of the problems and their practical utility have been kept in mind. It is believed that enough *new* ones have been introduced to awaken interest in the dullest boy.

It is intended that this work be supplemental to both text-books on Arithmetic and Geometry. Many text-books on Geometry are so abstract in treatment as to leave the pupil, even after completing his course, unable to apply what he has learned. Few persons can easily acquire the power of abstraction necessary for correct geometrical conceptions. Making the subject concrete, therefore, will be of great benefit to the majority of pupils. As soon as the pupil discovers the practical value of the propositions he demonstrates, he will be incited to study with new interest and zeal.

Fundamentals of Measurement, the frontispiece, can be made of much use to both teacher and pupil. It will be noticed that the fundamental quantities of measurement—points, lines, surfaces, and volumes—are so grouped as to picture with great vividness the relations of these fundamentals. Each block is independent of the others, and yet it requires all the blocks—and in the order given—to show systematically the essentials of measurement. By starting at the top of the monument and noticing in order the point, lines, plane surfaces, and solids, the synthetic process of reasoning becomes apparent. The simplicity of the arrangement is surpassed only by the comprehensiveness of the whole. By it the beginner can learn the elements, and the advanced student, at a glance, verify his conclusions.

The author tenders his thanks to Charles Lose, A.M., Superintendent of Schools of the city of Williamsport, Pa., the Hon. Emerson Collins, of the same place, and others, who have kindly given him valuable suggestions and criticisms during the progress of the work. He asks from his fellow teachers not only a hearty welcome, but criticisms as well.

S. W. FURST.

SUGGESTIONS TO TEACHERS

It is deemed best to show the application of the formula to those solids for which the rules usually given are short and simple, as well as to those for which the rules given are more complex. This is done by means of a chapter of illustrative problems, which can be used for reference for all following chapters on solids. It is believed that a plan for the solution of every problem in the book can be found by a study of this chapter. After the pupil thoroughly understands how to apply the formula to the solution of problems involving all the common solids, it is well to have him use in practice the shorter rules, especially for finding the volume of the prism, cylinder, etc.

The pupil should be thoroughly drilled on finding the area of the *middle section*. This can best be done by having him draw figures of different shapes and representing this section by dotted lines. In all cases he should write on each line the dimension given in the solid. Until he is able to do this preliminary work, it can not be expected that he will make rapid progress. The plan is made clear in the chapter of illustrative problems.

Constant reference should be made to the frontispiece. Many points often vague to the pupil can be made quite clear by a proper use of this. It is suggested that the pupil be taught to draw and letter this from memory, a block at a time, and finally the whole monument.

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MENSURATION

MENSURATION was formerly a part of the course of study of every well-ordered school of higher grade. The addition of a variety of subjects to the course of study gradually crowded it out, so that it often became merely an insignificant part of the arithmetic work.

Now a reaction has set in. The demand for thoroughness instead of variety is growing, and the great increase in "Industrial Education" has awakened an interest in mensuration as a practical subject and has called for increased attention to it.

A knowledge of mensuration is valuable to:

The housekeeper who buys wall-paper, curtains, carpets, etc.

The farmer who measures his fields and bins to count his crop values.

The carpenter who builds houses, barns, or shops.

The mason who builds walls and paves sidewalks.

The plumber who calculates pipe extension, water flow, and drainage.

The painter who paints buildings and fences and decorates interiors.

The contractor who paves streets and builds roads.

The cooper who makes kegs, casks, and barrels.

The landscape gardener who lays out grounds and flower beds.

The average man who employs artisans.

MENSURATION

INTRODUCTION

1. A **Brick** (Fig. 1) is a material body, since it occupies bounded space. The boundary of the brick is its *surface*. We can conceive the brick to be removed, and there would remain space formerly occupied by the brick; this is a *geometrical solid*. The surface bears the same relation as in the material body.

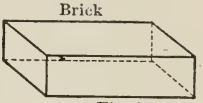


Fig. 1

2. A **Plane** is a surface such that a straight line connecting any two of its points lies wholly in its surface. Thus, MN (Fig. 2) is a plane, since all points of any straight line, as RS, lie wholly in its surface. Each face of the brick (Fig. 1) is a plane.

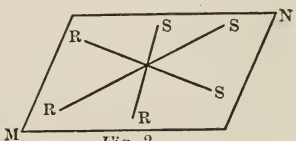


Fig. 2

3. When two planes intersect each other, their common intersection is a straight line. Each edge of the brick (Fig. 1) is a straight line, since in each case it is the intersection of two planes. RS (Fig. 3) is a straight line, it being the common intersection of the two planes AB and CD.

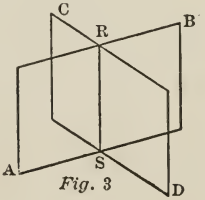


Fig. 3

4. A **Straight Line** is one which has the same direction at every point. AB (Fig. 4) is a straight line. When two straight lines intersect, their common intersection is a *point*. In Fig. 5 R is a point, it being the intersection of the two straight lines AB and CD.

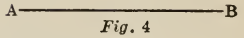


Fig. 4

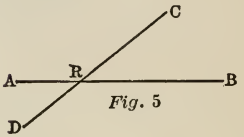


Fig. 5

5. A **Point** has no extension, but simply position.

A **Line** has only one dimension,— *length*.

A **Surface** has two dimensions,— *length* and *breadth*.

A **Solid,** or volume, has three dimensions,— *length*, *breadth*, and *thickness*.

The brick (Fig. 1) is a *solid*.

Any *face* of the brick is a *surface*.

Any *edge* of the brick is a *line*.

Any *corner* of the brick is a *point*.

A line is limited by a point; a surface is limited by a line; a volume is limited by a surface; and yet we can conceive of each of these as being independent of the other.

Points, lines, surfaces, and solids of geometry are purely imaginary.

The fundamental quantities of measurement are points, lines, surfaces, and volumes.

6. Mensuration is that branch of applied geometry which gives the rules and processes for finding the lengths of lines, the areas of surfaces, or the volumes of solids.

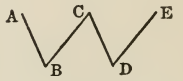
DEFINITIONS

LINES

7. A **Straight Line** is one which has the same direction at all points: as MN. It is the shortest distance between two points.

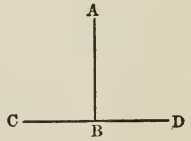
8. A **Curved Line** is one which changes its direction at every point: as RS.

9. A **Broken Line** is one which is composed of different successive straight lines: as ABCDE.



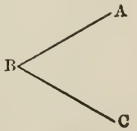
10. Parallel Lines are those in the same plane which have the same direction: as MN and ST. They can never meet, however far produced.

11. A **Perpendicular Line** is a straight line joined to another straight line, so as to incline equally to both sides. AB is perpendicular to CD.

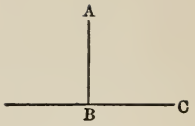


ANGLES

12. An **Angle** is the difference in direction between two lines which meet at a common point, called the vertex. ABC is an angle whose vertex is the point B, and whose sides are AB and BC.



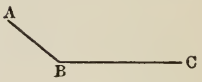
13. A **Right Angle** is an angle formed by two lines which are perpendicular to each other. ABC is a right angle.



14. An **Acute Angle** is one which is less than a right angle. MNO is an acute angle.

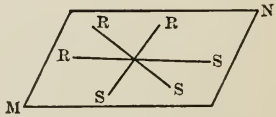
15. An **Obtuse Angle** is one which is greater than a right angle. ABC is an obtuse angle.

Acute and obtuse angles are *oblique* angles.



PLANE FIGURES

16. A **Plane** is a surface such that a straight line connecting any two of its points lies wholly in its surface. MN is a plane, since all points of the straight line RS lie wholly in its surface.



17. A **Plane Figure** is a portion of a plane surface bounded by straight or curved lines.

18. A **Polygon** is a plane figure bounded by straight lines; the lines are called the sides of the polygon.

19. The **Perimeter** of a polygon is the sum of its sides.

20. The **Area** of a plane figure is the surface included within the perimeter.

21. The **Diagonal** of a polygon is a line joining the vertices of any two of its angles not adjacent.

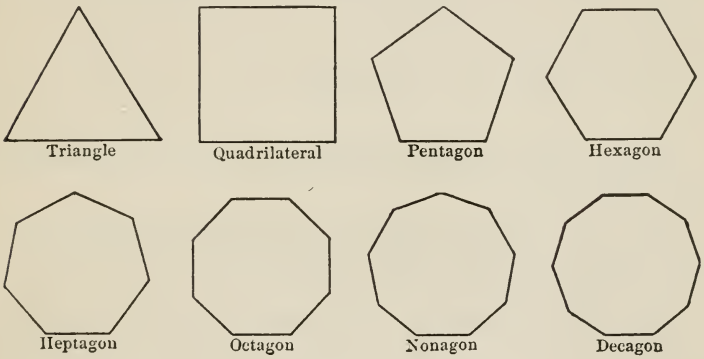
22. The **Altitude** of a polygon is the perpendicular distance between its base and a side or angle opposite.

23. An **Equilateral Polygon** has all its sides equal.

An **Equiangular Polygon** has all its angles equal.

A **Regular Polygon** is both equilateral and equiangular.

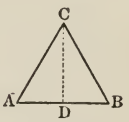
24. Polygons are named according to the number of sides. A polygon of three sides is a triangle; of four



sides, a quadrilateral; of five sides, a pentagon; of six sides, a hexagon; of seven sides, a heptagon; of eight sides, an octagon; of nine sides, a nonagon; of ten sides, a decagon; etc.

TRIANGLES

25. A **Triangle** is a plane figure having three sides and three angles. ABC is a triangle.



26. The **Base** of a triangle is the side on which it stands. AB is the base.

27. The **Altitude** of a triangle is a line drawn from the angle opposite perpendicular to the base,

or the base produced. CD is the altitude. The dotted lines in the following figures represent the *altitude*.

Fig. 1

Fig. 2

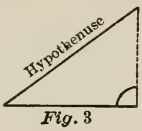


Fig. 3

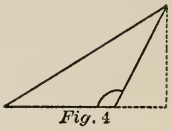


Fig. 4

Named by Sides	Equilateral	Isosceles		Scalene
Named by Angles	Equiangular			
	Acute-angled	Acute-angled	Right-angled	Obtuse-angled

28. A **Right-angled Triangle** is a triangle having one right angle.

29. The **Hypothenuse** of a right-angled triangle is the side opposite the right angle.

30. An **Equilateral Triangle** has its three sides equal.

31. An **Isosceles Triangle** has two of its sides equal.

32. A **Scalene Triangle** has no two sides equal.

33. An **Acute-angled Triangle** has three acute angles.

34. An **Obtuse-angled Triangle** has one obtuse angle.

35. An **Equiangular Triangle** has its three angles equal.

Rule I. To find the area of a triangle when the base and altitude are given: *Multiply the base by one-half the altitude.*

Rule II. To find the area of a triangle when the three sides are given: *Subtract each side separately from half the sum of the three sides; multiply the continued product of these three remainders by the half-sum; the square root of the product is the area.*

Rule III. To find one dimension of a triangle when the area and the other dimension are given: *Divide the area by one-half the given dimension.*

PROBLEMS

1. A triangle has a base of 60 feet, and an altitude of 20 feet; how many square feet does it contain?

Ans. 600 sq. ft.

2. Find the area of a triangle, the length of whose base is 50 feet, and whose altitude is 12 feet 4 inches.

Ans. $308\frac{1}{3}$ sq. ft.

3. How many acres in a triangular lot whose base is 28 rods and altitude 18 rods? Ans. 1 A. 92 sq. rds.

4. What is the cost of a triangular piece of land whose base is 30.96 chains and altitude 4.835 chains, at $120 an acre? Ans. $898.15—.

5. Find the area of a triangle whose sides are respectively 60, 80, and 100 feet. Ans. 2,400 sq. ft.

6. Find the base of a triangle whose area is 40 acres and altitude 80 rods. Ans. 160 rds.

7. A triangle contains 48 square feet 63 square inches; the base is 12 feet 6 inches; required the altitude. Ans. 7 ft. 9 in.

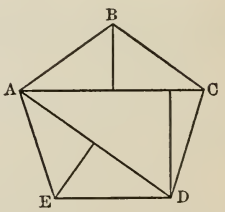
8. Find the cost of sodding a triangular plot of ground whose sides are respectively 60, 75, and 80 feet, at 15 cents per square yard. Ans. $35.60+.

9. Mr. A built a barn in the form of the letter L; the width of the barn at each end is 60 feet, and the ridge of the roof is 24 feet higher than the foot of the rafters; how many square feet of boards were required to cover the three gables? Ans. 2,160 sq. ft.

10. Owing to the difference in the soil of a farmer's field, which is square, he decided to plant part in corn and part in potatoes. He first made a straight furrow from the northeast corner of the field to the southwest corner, and found it to be 88 rods long; from the middle of this furrow to the southeast corner of the field is 44 rods. He planted all to the eastern side of the first furrow drawn in potatoes and the balance in corn. How many acres in the field? How many acres did he plant in potatoes? In corn?

Ans. field, $24\frac{1}{5}$ A.; potatoes, $12\frac{1}{10}$ A.; corn, $12\frac{1}{10}$ A.

11. Find the area of the pentagon as shown in the accompanying diagram. The diagonal AC is 48 inches, and the diagonal AD, 36 inches; the altitude of the triangle ABC is 16 inches, of the triangle ACD, 20 inches, and of the triangle AED, 12 inches. Ans. 1,080 sq. in.



THE RIGHT-ANGLED TRIANGLE

36. The following principles relating to right-angled triangles have been proved by Geometry. An examination of Figures 5, 6, and 7 will be helpful to the student in fixing them in mind.

PRINCIPLE I. *The square of the hypothenuse is equal to the sum of the squares of the other two sides.*

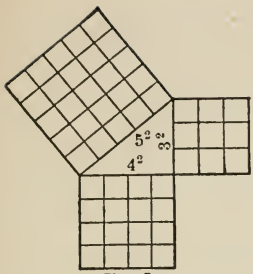


Fig. 5

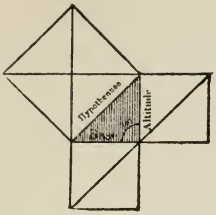


Fig. 6

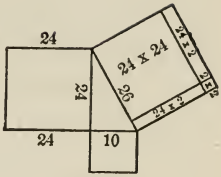


Fig. 7

PRINCIPLE II. *The square of the base, or of the perpendicular, is equal to the square of the hypothenuse diminished by the square of the other side.*

Rule I. To find the hypothenuse: *Extract the square root of the sum of the squares of the base and perpendicular.*

Rule II. To find the base or perpendicular: *Extract the square root of the difference between the square of the hypothenuse and the square of the given side.*

PROBLEMS

1. What is the length of the hypothenuse of a right-angled triangle (Fig. 5) whose base is 4 feet, and whose perpendicular is 3 feet?

OPERATION.—$\sqrt{4^2 + 3^2} = 5$ feet. Ans.

In a right-angled triangle, given:

2. The base 5 feet, perpendicular 12 feet, to find the hypothenuse. Ans. 13 ft.

3. The base 8 inches, perpendicular 15 inches, to find the hypothenuse. Ans. 17 in.

4. The perpendicular 20 rods, the base 21 rods, to find the hypothenuse. Ans. 29 rds.

5. The perpendicular 3 yards, the base 4 yards, to find the hypothenuse. Ans. 5 yds.

6. The hypothenuse 10 feet, the base 8 feet, to find the perpendicular. Ans. 6 ft.

7. The hypothenuse 78 feet, the perpendicular 30 feet, to find the base. Ans. 72 ft.

8. Find the diagonal of a square whose side is 20 feet. Ans. 28.28+ ft.

9. Find the diagonal of a cube whose edge is 20 feet. Ans. 34.64+ ft.

10. The gable end of a house 50 feet wide is 14 feet high; what is the length of the rafters? Ans. 28.65+ ft.

11. A flag-pole 120 feet high casts a shadow 110 feet in length; required the distance from the end of the shadow to the top of the pole. Ans. 162.78+ ft.

12. A hunter stood 40 feet from the foot of a tree and shot a squirrel on the top of a tree 55 feet high; how far did the bullet go before hitting the squirrel, provided the starting point of the bullet was 5 feet from the ground? Ans. 64.03+ ft.

13. Thomas Lighty, a carpenter, directed his apprentice to cut a brace of the proper length to secure a perpendicular in a sill, by measuring 6 feet each way from the corner where the two sticks joined; how long was the brace? Ans. 8.48+ ft.

14. A derrick at a granite quarry had an upright 48 feet long, and a horizontal arm 20 feet long; a rope ran on pulleys from the foot of the perpendicular to the extremity of the arm and thence to the ground; what length of rope was required, no allowance being made for fastening the rope? Ans. 100 ft.

15. A's horse travels a mile in 3 minutes, and B's a mile in 4 minutes; A drives at this rate due north for 24 minutes, and B, starting at the same place, due east for the same time; how far are they apart after driving 24 minutes? After driving at the same rate and same direction, respectively, 1 hour? Ans. 10 mi.; 25 mi.

16. Williamson Porter in hunting bees placed his bait on a stump 5 feet high; one bee flew in a "bee line" from the bait 75 feet to a tree due west from the stump, alighting 45 feet from the foot of the tree; another flew 80 feet east to a tree, alighting 65 feet from the foot of the tree; if the ground is level, and both trees perpendicular, what is the shortest distance between the trees? Ans. 116.35+ ft.

17. Mr. A, a politician, desired his three neighbors, B, C, and D, to vote at a certain election. A lived 100 rods south; B, 120 rods east; C, 140 rods north; and D, 150 rods west of the polling place. A, with his carriage, drove from home in a straight line for B, thence in a straight line for C, and thence in a straight line for D; from D's residence he drove in a straight line to the polling place, and thence in a straight line home: how far did he drive, provided he allowed his neighbors to walk home? Ans. 2 mi. 155.77+ rds.

QUADRILATERALS

37. A **Quadrilateral** is a portion of a plane bounded by four straight lines.

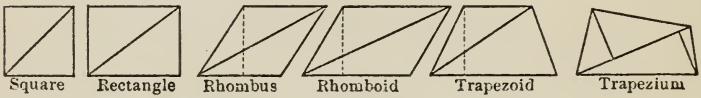
38. A **Parallelogram** is any plane figure of four straight sides, the opposite ones of which are parallel.

39. A **Rectangle** is any parallelogram having all its angles right angles.

40. A **Square** is a rectangle all of whose sides are equal.

41. A **Rhombus** is a parallelogram having all its sides equal and all its angles oblique.

42. A **Rhomboid** is a parallelogram whose opposite sides only are equal and whose angles are oblique.



Quadrilaterals { 1. Parallelograms { 1. Square, 2. Rectangle, 3. Rhombus, 4. Rhomboid
2. Trapezoid
3. Trapezium

43. A **Trapezoid** is a quadrilateral which has only two of its sides parallel.

44. A **Trapezium** is a quadrilateral having no sides parallel.

45. The **Lower Base** of a parallelogram is the side upon which it stands. The **Upper Base** is the side opposite the lower base.

46. The **Altitude** of a parallelogram or of a trapezoid is the perpendicular distance between two bases. The dotted lines in the figures represent the altitude.

47. The **Diagonal** of a quadrilateral is a straight line joining two opposite vertices.

Rule I. To find the area of any parallelogram: *Multiply the base by the altitude.*

Rule II. To find the area of a trapezoid: *Multiply half the sum of the parallel sides by the altitude.*

Rule III. To find the area of a trapezium: *Multiply the diagonal by half the sum of the perpendiculars drawn to it from the vertices of the opposite angles.*

NOTE.—The area of any polygon may be found by dividing it into triangles and finding the sum of their areas.

PROBLEMS

1. The side of a square is 20 feet; what is its area? Ans. 400 sq. ft.

2. A has a rectangular field 80 rods long and 60 rods wide; how many acres does it contain? Ans. 30 A.

3. Find the area of a parallelogram whose base is 150 feet and altitude 80.5 feet. Ans. 12,075 sq. ft.

4. Find the area of a rhombus whose base is 20 feet and altitude 6 feet 3 inches. Ans. 125 sq. ft.

5. A field in the form of a rhomboid has a base 136 rods long and an altitude of 550 yards; how many acres does it contain? Ans. 85 A.

6. The parallel sides of a trapezoid are 70 rods and 90 rods and the altitude 20 rods; find the area in acres. Ans. 10 A.

7. Find the area of a trapezium, a diagonal of which is 50 feet, and the perpendiculars of which to this diagonal are 10 feet and 35 feet. Ans. 1,125 sq. ft.

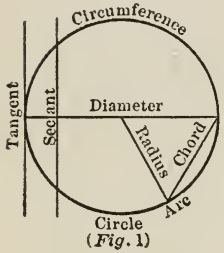
8. Required the area of a trapezium, the lengths of whose sides are respectively 20, 30, 25, and 35 chains, and the diagonal 40 chains, making two triangles whose sides are respectively 35, 20, and 40; and 30, 25, and 40.
Ans. 72 A. 71 sq. rds. 17 sq. yds. 5 sq. ft. 143.136 sq. in.

9. Required the area of a regular hexagon whose side is 6 inches. Ans. 93.528+ sq. in.

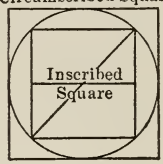
SUGGESTION.—Divide into 6 equilateral triangles, or 3 rhombuses, or 2 trapezoids.

THE CIRCLE

48. A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the center.

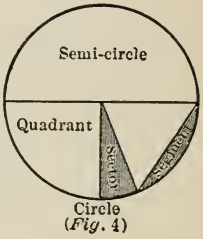


Circle (*Fig.* 1)



(*Fig.* 2)

(*Fig.* 3)



Circle (*Fig.* 4)

The Circle
- Lines
  - Curved
    1. Circumference
    2. Arc
  - Straight
    1. Diameter
    2. Radius
    3. Secant
    4. Tangent
    5. Chord
- Surfaces of portions of Circle
  1. Semi-circle
  2. Quadrant
  3. Sector
  4. Segment

49. The **Circumference** of a circle is the line which bounds the circle.

50. An **Arc** is any portion of the circumference.

51. The **Diameter** of a circle is a straight line passing through the center, and having its extremities in the circumference.

52. The **Radius** of a circle is a straight line drawn from the center to any part of the circumference. It is one-half the diameter.

53. A **Secant** is a straight line which cuts the circumference at two points.

54. A **Tangent** is a straight line which touches the circumference, but does not cut it.

55. A **Chord** is a straight line joining the extremities of an arc.

56. A **Semi-circle** is half a circle.

57. A **Quadrant** is a quarter of a circle.

58. A **Sector** of a circle is a portion of a circle included by two radii and the arc intercepted by them.

59. A **Segment** of a circle is a portion of a circle included between an arc and its chord.

60. The ratio of the circumference to the diameter is the same for all circles. For convenience this ratio is represented by the Greek letter π (pi). The approximate numerical value of π is 3.1416, which is the value used in this work.

Rule I. To find the circumference of a circle when the diameter is given: *Multiply the diameter by 3.1416.*

Rule II. To find the diameter of a circle when the circumference is given: *Divide the circumference by 3.1416.*

Rule III. To find the area of a circle when its diameter, radius, and circumference are given, or when either is given:

(a) *Multiply the circumference by one-fourth of its diameter;* or,

(b) *Multiply the square of the diameter by .7854;* or,

(c) *Multiply the square of the radius by 3.1416;* or,

(d) *Multiply half the distance round by half the distance through.*

Rule IV. To find either dimension, when the area is given:

Let A represent the area of a circle, C the circumference, D the diameter, and R the radius; then,

$$D = \sqrt{\frac{A}{.7854}}; \quad C = (\sqrt{\frac{A}{.7854}}) \times 3.1416; \quad R = \sqrt{\frac{A}{3.1416}}.$$

Rules I, II, and III may be stated as follows:

$$C = \pi D \text{ or } 2\pi R; \quad D = \frac{C}{\pi}; \quad A = \pi R^2 \text{ or } \frac{\pi D^2}{4}.$$

Rule V. To find the side of an inscribed square when the diameter is given:

(a) *Extract the square root of one-half the square of the diameter;* or,

(b) *Multiply the diameter by .7071.*

Rule VI. To find the area of a circular ring formed by two concentric circles: *Find the areas of the circles separately and take their difference.*

PROBLEMS

1. Find the circumference of a circle whose diameter is 20 inches. Ans. 62.832 in.

2. Find the circumference of a circle whose diameter is 39 feet. Ans. 122.522 ft.

3. Find the diameter of a circle whose circumference is 128.8056 yards. Ans. 41 yds.

4. Find the diameter of a circle whose circumference is 37.6992 inches. Ans. 12 in.

5. Find the radius of a circle whose circumference is 119.3808 inches. Ans. 19 in.

6. Find the radius of a circle whose circumference is 100.5312 rods. Ans. 16 rds.

7. How far is it round a tree which measures 4 feet over the stump? Ans. 12.5664 ft.

8. The spokes of the fore wheel of a buggy are 2 feet long, and of the hind wheel 2 feet 2 inches; no allowance being made for the thickness of the hub, how much longer is the tire of the hind wheel than of the fore wheel? Ans. 1.0472 ft.

9. Four flower-pots each with a lower diameter of 1 foot were placed on a bench 10 feet long and 16 inches wide; how much of the bench remained uncovered? Ans. 1467.6096 sq. in.

10. John threw a base-ball 2 inches in diameter at a paste-board card 2 feet square, and made in the card 15 holes of the same circumference as the ball; how much of the surface of the card remained unbroken? Ans. 528.876 sq. in.

11. Certain trustees asked for bids to fresco the ceiling of their church, which is 60 feet long and 40 feet wide. A contractor offered to place on the ceiling 8 circular designs 12 feet in circumference at $3 each, and paint the remaining surface at 9 cents per square yard, *or* to place on the ceiling 8 square designs, the perimeter of each to be 12 feet, at $3 each, and paint the remaining surface at 1 cent per square foot. The trustees chose the second offer. How much did it cost them? How much did they save or lose by taking the second offer instead of the first? Ans. cost, $47.28; lost, 20c.

12. The minute hand of a town clock is 2 feet long; over how much surface does it pass in 20 minutes? In 15 minutes? Ans. 4.1888 sq. ft.; 3.1416 sq. ft.

13. How many times will a wheel whose radius is 2 feet revolve in going 1 mile? Ans. 420.168+ times.

14. Find the width of the ring between two concentric circles whose circumferences are respectively 15.708 and 31.416 feet. Ans. $2\frac{1}{2}$ ft.

15. Find the area of a circle inscribed in a square containing 576 square feet. Ans. 452.39+ sq. ft.

16. Find the area of a circle circumscribed about a square containing 256 square feet. Ans. 402.109+ sq. ft.

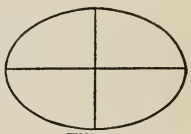
17. The diameter of a circle is 100 feet; find the side of the inscribed square. Ans. 70.71 ft.

18. The circumference of a circle is 62.832 feet; find the side of an inscribed square. Ans. 14.142 ft.

19. The area of a circle is 1,963.44 square inches; find the side of an inscribed square. Ans. 35.355 in.

THE ELLIPSE

61. An **Ellipse** is a curved plane formed by an oblique section of either a cone or a cylinder, passing through its curved surface, but not touching its base; the sum of the distances from every point of the bounding line to two fixed points, called foci, is equal to a line drawn through these points and terminated by the curve. The line drawn through the foci is a *transverse* axis; a perpendicular to this axis at its middle point is the *conjugate* axis.


Ellipse

Rule. To find the area of an ellipse: *Find the product of the semi-axes, and multiply this product by 3.1416.*

PROBLEMS

1. What is the area of an ellipse whose transverse axis is 40 inches and conjugate axis 32 inches?
Ans. 1,005.312 sq. in.

2. The axes of an elliptical flower bed are 20 and 16 feet; what is its area? Ans. 251.328 sq. ft.

3. The transverse axis of an elliptical fish pond is 80 feet and conjugate axis 60 feet; find the area.
Ans. 3,769.92 sq. ft.

4. How much larger is a circle whose diameter is 10 inches than an ellipse whose transverse axis is equal to the diameter of the circle, and conjugate axis $\frac{3}{5}$ of the length of the transverse axis? Ans. 31.416 sq. in.

SIMILAR PLANE FIGURES

62. Similar Plane Figures are those which have the same form. Their angles are equal and their like dimensions proportional. Squares, equiangular triangles, regular polygons of the same number of sides, and circles are similar. The surfaces of similar solids are also proportional. The like dimensions of circles are their radii, diameters, and circumferences, and of other plane figures, those which are like placed. The following principles are derived from Geometry:

PRINCIPLE I. *The areas of similar surfaces are to each other as the squares of their like dimensions.*

PRINCIPLE II. *The like dimensions of similar surfaces are to each other as the square roots of their areas.*

PROBLEMS

1. How many circles each 8 inches in diameter will equal the area of a circle 4 feet in diameter? Ans. 36.

2. John's slate is 14 by 10 inches, and William's 7 by 5 inches; how do they compare in size?
Ans. John's = 4 times William's.

3. Two farms of exactly the same shape contain respectively 144 and 169 acres. One side of the former is 480 rods; required the corresponding side of the latter. Ans. 520 rds.

4. A has a square field, the side of which is 40 rods, worth $50 an acre, and B a square field, $\frac{1}{2}$ as long, worth $100 an acre; which is the more valuable, and how much? Ans. A's, $250 more.

5. If it cost $40 to carpet a rectangular room 20 feet long, how much will it cost to carpet a room of similar shape 25 feet long? Ans. $62.50.

6. A lady has two circular flower beds; the one $6\frac{1}{2}$ feet in diameter, the other $3\frac{1}{4}$ feet; the first is how many times the size of the second? Ans. 4 times.

7. The area of a rectangular field is 2,880 square rods; its sides are as 4 to 5; what is the length of each side? Ans. 48 rds.; 60 rds.

8. The hypothenuse of a right-angled triangle is 25 inches; what is the hypothenuse of a similar triangle which contains twice the area? Ans. 35.35+ in.

9. The area of a trapezoid is 216 square rods, and its altitude 12 rods; find the altitude of a similar trapezoid whose area is 384 square rods. Ans. 16 rds.

10. A battle ship whose anchor weighs 4,000 pounds requires a cable 3 inches in diameter; what should be the diameter of the cable when the anchor weighs 4 tons? Ans. 4.242+ in.

11. If a cistern can be filled by a pipe 3 inches in diameter in 24 minutes, in what time can it be filled by a pipe 9 inches in diameter? Ans. 2 min. 40 sec.

12. How many saucers each 5 inches in diameter will equal in area the surface of a round table 5 feet in diameter? Ans. 144.

13. If the circumference of a circle is 2 feet, what is the circumference of a circle $3\frac{1}{16}$ times as great?

Ans. $3\frac{1}{2}$ ft.

14. The altitudes of two similar triangles are 21 feet and $5\frac{1}{4}$ feet; what is the relation of their areas? Ans. 16.

15. The area of a triangle is 5,400 square feet, and its sides are proportional to the numbers 9, 12, and 15; required the length of the sides.

Ans. 90 ft., 120 ft., 150 ft.

16. Mr. Bowers wished to sow clover seed on two fields, one 40 rods square and the other 60 rods square; he estimated that one bushel would be sufficient for the smaller field; what should have been his estimate for the larger field? Ans. $2\frac{1}{4}$ bu.

17. Sarah baked for John 2 buckwheat cakes each 6 inches in diameter, and for Rufus 8 cakes of the same thickness each 3 inches in diameter; John accused Sarah of unfairness; was she unfair?

18. If it requires 96 square inches of tissue paper to cover a cube whose edge is 4 inches, how much would be required to cover a cube whose edge is 8 inches?

Ans. 384 sq. in.

19. It cost Mr. McDowell \$80 to pave his walk, which is 6 feet wide and 100 feet long; Mr. Blair paid \$320 for paving his walk similar in shape to Mr. McDowell's, with the same material; what are the dimensions of Mr. Blair's walk? Ans. 12 ft. wide; 200 ft. long.

MENSURATION OF SOLIDS

PRISMS AND CYLINDERS

63. A **Solid** or **Body** has three dimensions,—length, breadth, and thickness. The bounding planes are its faces, and their intersecting lines its edges.

64. A **Prism** is a solid whose two ends are parallel, similar, and equal, and whose sides are parallelograms.

They are named according to the form of their bases; as triangular, quadrangular, pentagonal, hexagonal, etc.

65. The **Altitude** of a prism is the perpendicular distance between its bases. In Figs. 4, 5, and 6 the perpendicular dotted lines represent the altitude.

66. A **Right Prism** is a prism whose lateral edges are perpendicular to the bases.

67. An **Oblique Prism** is a prism whose lateral edges are not perpendicular to the bases.

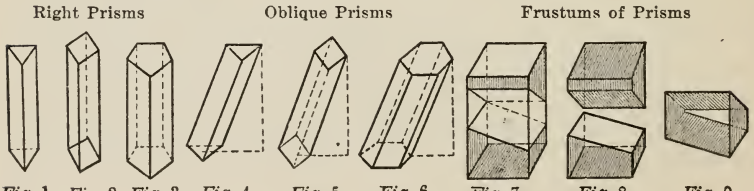


Fig. 1 *Fig.* 2 *Fig.* 3 *Fig.* 4 *Fig.* 5 *Fig.* 6 *Fig.* 7 *Fig.* 8 *Fig.* 9

68. A **Frustum** of a prism is that part which remains after cutting off a part of it by passing a plane through it not parallel to either base. This is also called a *truncated prism*.

69. A **Parallelopiped** is a prism bounded by six parallelograms, the opposite faces being parallel.

70. A **Right Parallelopiped** is one whose lateral edges are perpendicular to its bases.

PARALLELOPIPEDS

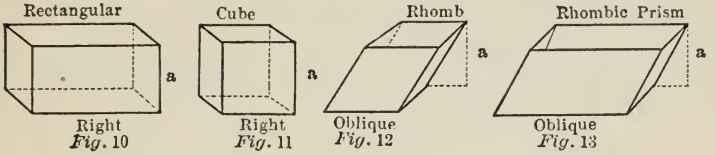


Fig. 10 *Fig.* 11 *Fig.* 12 *Fig.* 13

71. A **Rectangular Parallelopiped** is a right parallelopiped whose bases are rectangles. All of its faces are rectangles.

72. A **Cube** is a parallelopiped whose faces are all equal squares.

73. A **Rhomb** is a parallelopiped whose faces are all equal rhombuses.

74. A **Rhombic Prism** is a parallelopiped, whose faces are rhombuses or rhomboids, having each pair of *opposite* faces equal, but not all its faces equal. In each figure above, a represents the altitude.

75. A **Cylinder** is a solid bounded by a uniformly curved surface, and having for its ends two equal parallel circles. Any section of a cylinder parallel to the ends is a circle equal to either end.

76. A **Right Cylinder** is one whose ends are perpendicular to its sides.

77. An **Oblique Cylinder** is one whose ends are not perpendicular to its sides.

78. The **Bases** of a cylinder are the two equal and parallel circles.

79. The **Altitude** of any cylinder is the perpendicular distance between the planes of its bases.

80. The **Axis** of a cylinder is a line joining the centers of its bases.

81. A **Cylindroid** is a solid resembling a cylinder, but having its bases elliptical.

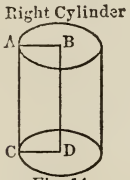


Fig. 14

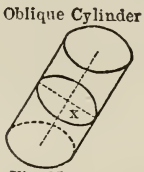


Fig. 15

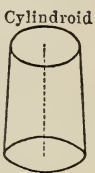


Fig. 16

The right cylinder (Fig. 14) may be conceived to be generated by the revolution of the rectangle ABCD about its side BD, as an axis.

A *right section* of a prism or a cylinder is a section made by a plane perpendicular to the axis; as X (Fig. 15).

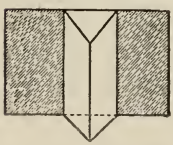


*Fig.*17

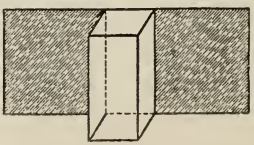


*Fig.*18

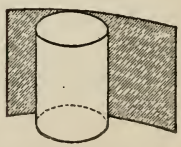


*Fig.*19

82. By examining Figs. 17, 18, and 19, it will be observed that the *convex* surface of each may be represented by a rectangle, the perimeter of the prism or cylinder being

equal to the base of the rectangle, and the altitude of the prism or cylinder equal to the altitude of the rectangle; hence the following rule:

Rule. To find the *convex* surface of a right prism or right cylinder: *Multiply the perimeter of the base by the altitude.*

For the *entire* surface, add to the convex surface the areas of the two bases.

The convex surface of *any* prism may be found by adding together the areas of the lateral faces.

The convex surface of *any* cylinder may be found by finding the product of the perimeter of a right section of the cylinder and the axis.

SURFACES OF PRISMS AND CYLINDERS

PROBLEMS

1. What is the convex surface of a prism whose altitude is 10 feet and perimeter of the base 20 feet?

Ans. 200 sq. ft.

2. What is the convex surface of a prism whose altitude is 10 yards and perimeter of the base 10 yards?

Ans. 100 sq. yds.

3. What is the convex surface of a hexagonal prism whose altitude is 20 feet and each side of the base 12 inches? Ans. 120 sq. ft.

4. What is the convex surface of a cylinder, the circumference of whose base is 32.66 feet and altitude 10 feet?

Ans. 326.6 sq. ft.

5. What is the convex surface of a cylinder whose altitude is 20 feet and the radius of the base 5 feet?

Ans. 628.32 sq. ft.

6. The diameter of the base of a cylinder is 20 feet and the altitude 30 feet; required the convex surface.

Ans. 1,884.96 sq. ft.

7. Find the entire surface of a quadrangular prism whose altitude is 20 feet and each side of the base 3 feet.

Ans. 258 sq. ft.

8. What is the convex surface of a pentangular prism whose altitude is 30 feet and each side of the base 4 feet?

Ans. 600 sq. ft.

9. What is the convex surface of a cylinder whose altitude is 4 yards and diameter of the base 3 feet?

Ans. 113.0976 sq. ft.

10. The altitude of a triangular prism is 5 inches and each side of the base 6 inches; what is the entire surface? Ans. 121.1768 sq. in.

11. Required the entire surface of a parallelopiped 8 feet long, 5 feet wide, and 3 feet high. Ans. 158 sq. ft.

12. What is the entire surface of a cube whose edge is 7 feet? Ans. 294 sq. ft.

13. What is the entire surface of a rhomb, each face of which is a parallelogram, whose base is 2 feet 2 inches and altitude 2 feet? Ans. 26 sq. ft.

14. A door 8 feet high and 4 feet wide revolves around one of its sides as an axis; what is the convex surface of the cylinder generated by it? Ans. 201.0624 sq. ft.

15. A door 8 feet long and 4 feet wide revolves upon a point in its center; what is the convex surface of the cylinder generated by it; what is the entire surface?

Ans. 100.5312 sq. ft.; 125.664 sq. ft.

16. The diameter of the base of a cylinder is 16 inches, and the altitude $2\frac{1}{2}$ feet; how many square feet are there in the lateral surface? Ans. 10.472 sq. ft.

ILLUSTRATIVE PROBLEMS

Solved by the Prismoidal Formula

83. The special feature of this work is the application of the Prismoidal Formula to finding the volume of solids. Since this one rule covers so broad a field of measurement, it is thought that a thorough knowledge of the method of using it in finding the volume of the cube, cylinder, etc., as well as of those solids for which the rules usually given are more complex, will be beneficial. The author does not advise the teacher to discard the well known rules for finding the volume of a prism, cylinder, etc.; but he does recommend that this rule be substituted for those usually quoted for finding the volume of many other solids, as the frustum of a pyramid or of a cone. Both rules are given under each subject; this will enable the teacher in each case to take his choice. In order that the pupil may be made quite familiar with the rule, a number of illustrative problems are solved, and such suggestions made as will enable the pupil to use it intelligently in following chapters. These are arranged in a separate section at this place so that comparisons may be more advantageously made. Future reference is made to this chapter. For a derivation of the formula, etc., see Supplement.

Rule.—To find the volume of a prismoid, etc.:

84. *Multiply the sum of both end areas and four times the area of a section half way between them, by one-sixth the altitude.*

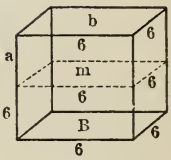
Letting a = the altitude or length; B, the area of either base, and, when unequal, the larger base; b, the area of the opposite

base, and, when unequal, the smaller base; and 4m, four times the area of a section half way between the bases, the rule may be stated as follows: $\frac{a}{6}(B + b + 4m)$.

This formula will be used throughout this work.

In all figures in the following chapter, the middle section is represented by dotted lines and the altitude is marked with the letter a.

1. Required the volume of a cube whose edge is 6 feet.

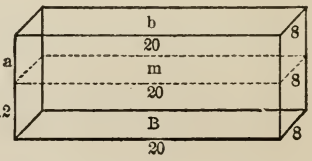


OPERATION.— $B = 6 \times 6 = 36$ sq. ft.
$b = 6 \times 6 = 36$ "
$4m = 4\ (6 \times 6) = 144$ sq. ft.
$a = 6$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{6}{6}(36 + 36 + 144) = 216$ cubic feet, volume.

REMARK.— It will be observed that in all uniform solids not tapering or having an apex, such as prisms and cylinders, the area of both bases and of *once* the middle section is the same; this necessitates only one calculation for these areas.

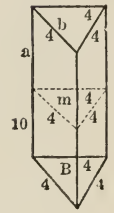
2. Required the number of cubic feet of air in a room 20 feet long, 8 feet wide, and 12 feet high.



OPERATION.—
$B = 20 \times 8 = 160$ sq. ft.
$b = 20 \times 8 = 160$ "
$4m = 4\ (20 \times 8) = 640$ sq. ft.
$a = 12$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{12}{6}(160 + 160 + 640) = 1{,}920$ cubic feet.

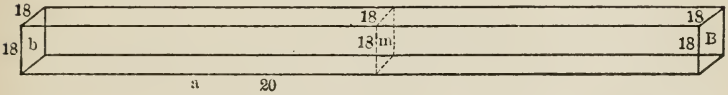
3. What is the volume of a triangular prism whose altitude is 10 feet, and each side of the base 4 feet?



OPERATION. — B = 6.928 sq. ft. (Art. 35, R. II)
b = 6.928 " " "
4m = 27.712 " " "
a = 10 " " "

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{10}{6}(6.928 + 6.928 + 27.712) = 69.28$ cubic feet, volume.

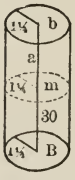
4. What is the cost of a piece of timber 18 inches square and 20 feet long, at $.30 a cubic foot?



OPERATION. — 18 inches = $\frac{3}{2}$ ft.
B = $\frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$ sq. ft.
b = $\frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$ "
4m = $4(\frac{3}{2} \times \frac{3}{2}) = 9$ sq. ft.
a = 20

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{20}{6}(\frac{9}{4} + \frac{9}{4} + 9) = 45$ cubic feet, volume.
$45 \times \$.30 = \$13.50$, cost.

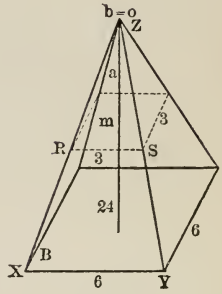
5. What is the volume of a cylinder whose radius is 1 foot 3 inches, and altitude 30 feet?



OPERATION. — 1 foot 3 inches = $\frac{5}{4}$ ft.
B = $(\frac{5}{4})^2 \times 3.1416 = 4.90875$ sq. ft. (Art. 60, R. III c)
b = $(\frac{5}{4})^2 \times 3.1416 = 4.90875$ " " "
4m = $4(\frac{5}{4})^2 \times 3.1416 = 19.635$ " " "
a = 30

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{30}{6}(4.90875 + 4.90875 + 19.635) = 147.2625$ cubic feet, volume.

6. What is the volume of a pyramid whose base is 6 feet square, and altitude 24 feet?



OPERATION.—In the triangle X Y Z, $\overline{6+0} \div 2$ or 3 is the length of the line RS, or the side of the middle section.

$B = 6 \times 6 = 36$ sq. ft.

$b = 0$

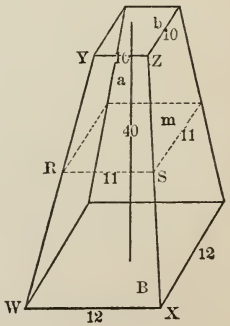
$4m = 4\ (3 \times 3) = 36$ sq. ft.

$a = 24$

Substituting in formula, $\frac{a}{6}\ (B + b + 4m)$ we have, $\frac{24}{6}\ (36 + 0 + 36) = 288$ cubic feet, volume.

REMARKS.—(*a*) It is apparent that in the pyramid and cone the area of the smaller base is Zero. (*b*) Since similar surfaces are to each other as the squares of their like dimensions, and since in the pyramid and cone the *side* or *diameter* of the lower base is *twice* the length of the side or diameter of the middle section, it follows that the area of the larger base is equal to *four times* the area of the middle section. Therefore, only one calculation is necessary for both areas.

7. Find the volume of the frustum of a square pyramid whose altitude is 40 feet, each side of the lower base 12 feet, and of the upper base 10 feet.



OPERATION.—The face W X Y Z, is a trapezoid. Therefore, $\frac{1}{2}$ the sum of the parallel sides, or $\frac{1}{2}\ (12 + 10)$ or 11 feet is the length of the line RS, the side of the middle section.

$B = 12 \times 12 = 144$ sq. ft.

$b = 10 \times 10 = 100$ "

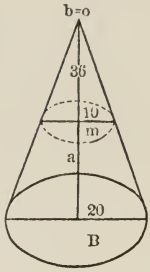
$4m = 4\ (11 \times 11) = 484$ sq. ft.

$a = 40$

Substituting in formula, $\frac{a}{6}\ (B + b + 4m)$ we have, $\frac{40}{6}\ (144 + 100 + 484) = 4{,}853\frac{1}{3}$ cubic feet, volume.

N. B.—By this rule no root is extracted.

8. Find the volume of a cone, the diameter of whose base is 20 feet, and altitude 36 feet.



OPERATION.—$\overline{20 + 0} \div 2 = 10$, diameter of middle section.

$B = 10^2 \times 3.1416 = 314.16$ sq. ft. (Art. 60, R. III c)

$b = 0$

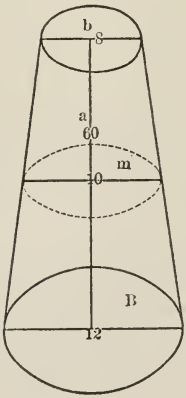
$4m = 4(5^2 \times 3.1416) = 314.16$ " " "

$a = 36$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$

we have, $\frac{36}{6}(314.16 + 0 + 314.16) = 3{,}769.92$ cubic feet, volume.

9. The altitude of the frustum of a cone is 60 feet, the diameter of the lower base 12 feet, and of the upper base 8 feet; required the volume.



OPERATION.— $\overline{12 + 8} \div 2 = 10$, diameter of middle section.

$B = 6^2 \times 3.1416 = 113.0976$ sq. ft. (Art. 60, R. III c)

$b = 4^2 \times 3.1416 = 50.2656$ " " "

$4m = 4(5^2 \times 3.1416) = 314.16$ " " "

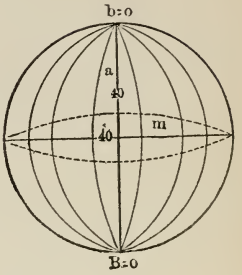
$a = 60$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$

we have, $\frac{60}{6}(113.0976 + 50.2656 + 314.16) =$ 4,775.232 cubic feet, volume.

From the principles of Geometry we have the following: The volume of the frustum of a cone is equal to the sum of the volumes of three cones, having for a common altitude the altitude of the frustum, and for bases the two bases of the frustum and a mean proportional between them. This involves the extraction of a root. In the above solution no root is extracted.

10. Find the volume of a sphere whose diameter is 40 inches.

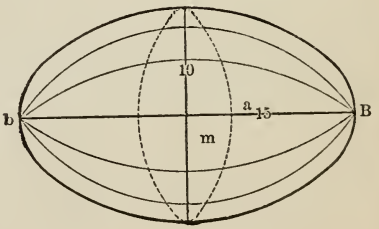


OPERATION. — In the sphere any diameter may be regarded as the altitude, and the area of any great circle, the area of a middle section. The area of each base is Zero.

$B = 0$
$b = 0$
$4m = 4\,(20^2 \times 3.1416) = 5{,}026.56$ sq. ft. (Art. 60, R. III c)
$a = 40$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{40}{6}(0 + 0 + 5{,}026.56) = 33{,}510.4$ cubic inches, volume.

11. What is the volume of a prolate spheroid whose shorter axis is 10 feet and longer axis 15 feet?



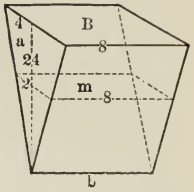
OPERATION. — Regard the longer axis as the altitude, and the shorter axis as the diameter of the middle section. The area of each base is Zero.

$B = 0$
$b = 0$
$4m = 4\,(5^2 \times 3.1416) = 314.16$ sq. ft. (Art. 60, R. III c)
$a = 15$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{15}{6}(0 + 0 + 314.16) = 785.4$ cubic feet, volume.

REMARK. — The problem can also be solved by using the shorter axis as the altitude, and finding the area of the elliptical middle section.

12. Required the volume of a wedge, the flat end of which is a rectangle 4 by 8 inches, and the height 24 inches.



OPERATION.—$B = 4 \times 8 = 32$ sq. in.

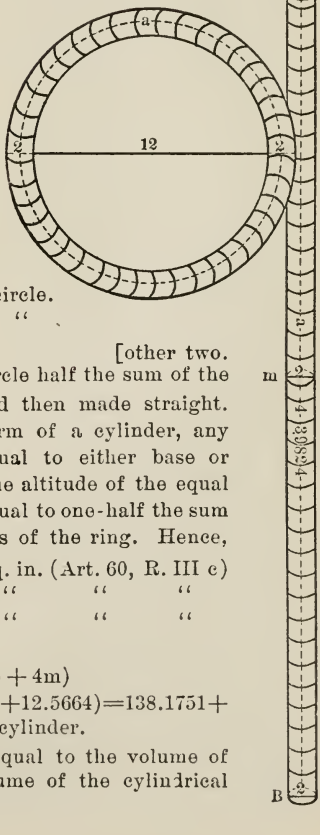
$b = 0$

$4m = 4\,(2 \times 8) = 64$ sq. in.

$a = 24$

Substituting in formula, $\frac{a}{6}\,(B + b + 4m)$ we have, $\frac{24}{6}\,(32 + 0 + 64) = 384$ cubic inches, volume.

13. Required the contents of a cylindrical ring, the thickness of which is 2 inches, and the inner diameter 12 inches.



OPERATION. — $12 + 2 + 2 = 16$ inches, diam. of outer circle.

$16 \times 3.1416 = 50.2656$ in. cir. outer circle.

$12 \times 3.1416 = 37.6992$ " " inner " "

$2\overline{)87.9648}$ [other two.

43.9824 " of a circle half the sum of the

Conceive the ring to be cut and then made straight. It is now an equal solid in the form of a cylinder, any cross section of the ring being equal to either base or middle section of the cylinder, and the altitude of the equal solid in the cylindrical form being equal to one-half the sum of the inner and outer circumferences of the ring. Hence,

$B = 1^2 \times 3.1416 = 3.1416$ sq. in. (Art. 60, R. III c)

$b = 1^2 \times 3.1416 = 3.1416$ " " "

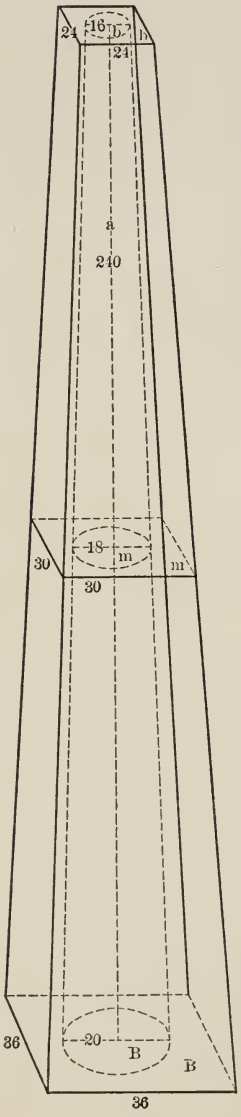
$4m = 4\,(1^2 \times 3.1416) = 12.5664$ " " "

$a = 43.9824$

Substituting in formula, $\frac{a}{6}\,(B + b + 4m)$

we have, $\frac{43.9824}{6}\,(3.1416 + 3.1416 + 12.5664) = 138.1751+$ cubic inches, volume of the cylinder.

The volume of the cylinder is equal to the volume of the cylindrical ring; hence, the volume of the cylindrical ring equals 138.1751+ cubic inches.



14. A certain monument has the form of the frustum of a square pyramid; how many cubic feet of material does it contain if it is 240 feet high, the lower base 36 feet square, and the upper base 24 feet square, provided that through the center, from top to bottom, there is an opening in the form of a frustum of a cone whose lower diameter is 20 feet, and upper diameter 16 feet?

OPERATION.—*Frustum of pyramid:*

$$B = 36 \times 36 = 1{,}296 \text{ sq. ft.}$$
$$b = 24 \times 24 = 576 \text{ "}$$
$$4m = 4(30 \times 30) = 3{,}600 \text{ "}$$

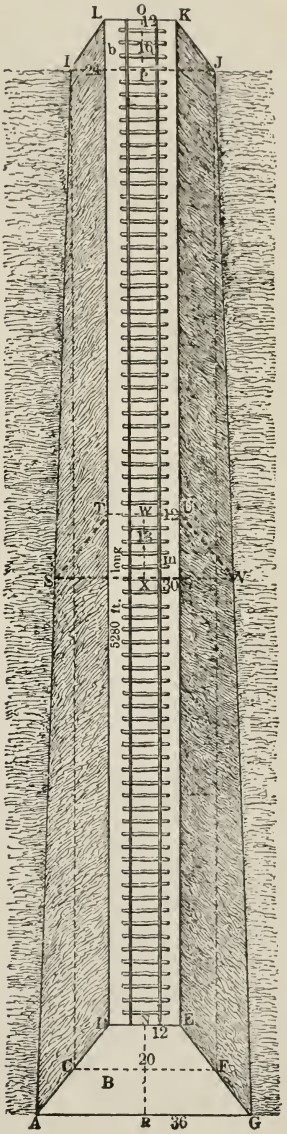
Substituting in formula, $\frac{a}{6}(B + b + 4m)$ we have, $\frac{240}{6}(1{,}296 + 576 + 3{,}600) = 218{,}880$ cubic feet volume if there were no opening.

Frustum of cone:

$$20 \div 2 = 10 \text{ R of B.}$$
$$16 \div 2 = 8 \text{ R of b.}$$
$$18 \div 2 = 9 \text{ R of M.}$$
$$B = 10^2 \times 3.1416 = 314.16 \text{ sq. ft.}$$
$$b = 8^2 \times 3.1416 = 201.0624 \text{ "}$$
$$4m = 4(9^2 \times 3.1416) = 1{,}017.8784 \text{ sq. ft.}$$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$ we have, $\frac{240}{6}(314.16 + 201.0624 + 1{,}017.8784) = 61{,}324.032$ cubic feet in frustum of cone to be deducted. Hence, 218,880 — 61,324.032 = 157,554.968 cubic feet of material.

15. Mr. Brown contracted to build an embankment for a railroad up a slightly inclined plane, the bed for the track to be level. The embankment was to be 1 mile long, and the width of the bed for the ties 12 feet wide; the perpendicular measurement of the higher end was to be 20 feet, and at the lower end 16 feet; the width of the embankment at the higher end at the bottom was to be 36 feet, and at the lower end 24 feet; what was the cost at 20 cents per cubic yard?



OPERATION.—ADEG, IJKL, and STUV, which are B, b, and M, are all trapezoids. Hence,

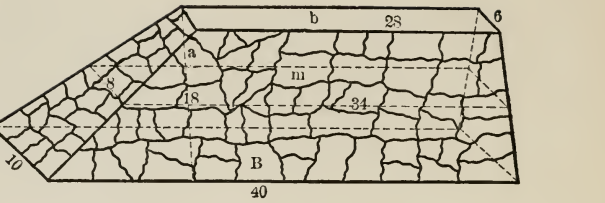
$$B = (\overline{36+12} \div 2)20 = 480 \text{ sq. ft.}$$

$$b = (\overline{24+12} \div 2)16 = 288 \text{ "}$$

$$4m = 4\{(\overline{30+12} \div 2)18\} = 1{,}512 \text{ "}$$

$$a = 5{,}280 \text{ ft.}$$

Substituting in formula, $\frac{a}{6}(B+b+4m)$ we have, $\frac{5280}{6}(480+288+1{,}512) =$ 2,006,400 cubic feet. 2,006,400 $\div$ 27 = 74,311$\frac{1}{9}$ cubic yards. 74,311$\frac{1}{9} \times$ 20 cents = \$14,862.22$\frac{2}{9}$ cost.



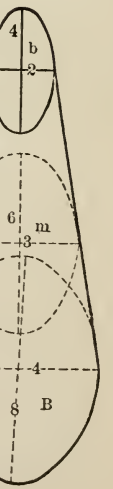
16. A bridge-pier of solid masonry has a rectangular base 10 by 40 feet; the top of the pier is a rectangle 6 by 28 feet, and the perpendicular distance between the two bases is 18 feet; how many perches of material in the pier?

OPERATION.—

$\overline{40 + 28} \div 2 = 34$ ft. side of middle section.
$\overline{10 + 6} \div 2 = 8$ ft. end " " "
$B = 40 \times 10 = 400$ sq. ft.
$b = 28 \times 6 = 168$ "
$4m = 4\,(34 \times 8) = 1{,}088$ "
$a = 18$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{18}{6}(400 + 168 + 1{,}088) = 4{,}968$ cubic feet.
$4{,}968 \div 24.75 = 200.72+$ perches, Ans.

17. A saddler uses a block for making horse collars shaped like a cylindroid. Find the volume of such a block whose lower base is an ellipse having a transverse axis 8 inches long, conjugate axis 4 inches long, and whose upper base is an ellipse, the transverse axis being 4 inches long, the conjugate axis 2 inches long, if the altitude of the block is 30 inches.



OPERATION.—

$$\begin{aligned} B &= 4 \times 2 \times 3.1416 = 25.1328 \text{ sq. in. (Art. 61. R)} \\ b &= 2 \times 1 \times 3.1416 = 6.2832 \quad \text{“} \quad \text{“} \\ 4m &= 4(3 \times 1\tfrac{1}{2} \times 3.1416) = 56.5488 \quad \text{“} \quad \text{“} \\ a &= 30 \end{aligned}$$

Substituting in formula, $\frac{a}{6}(B + b + 4m)$
we have, $\frac{30}{6}(25.1328 + 6.2832 + 56.5488) = 439.824$ cubic inches, Ans.

$\overline{8+4} \div 2 = 6$ transverse axis of M.
$\overline{4+2} \div 2 = 3$ conjugate “ “

VOLUMES OF PRISMS AND CYLINDERS

85. Rule I. To find the volume of a prism or cylinder: *Multiply the area of the base by the altitude.*

Rule II. *Multiply the sum of both end areas and four times the area of a section half-way between them, by one-sixth the altitude.* $\frac{a}{6}(B + b + 4m)$.

PROBLEMS

In a prism or cylinder, given:

1. Side of square base 4 feet, altitude 30 feet, to find the volume. Ans. 480 cu. ft.

2. Side of square base $3\frac{1}{2}$ feet, altitude 8 feet, to find the volume. Ans. 98 cu. ft.

3. Side of regular hexagonal base 8 inches, altitude 10 feet, to find the volume. Ans. 11.546+ cu. ft.

4. Sides of a triangular base 12, 15, and 24 feet, altitude 20 feet, to find the volume. Ans. 1,472.676+ cu. ft.

5. Diameter of the base 4 feet, altitude 30 feet, to find the volume. Ans. 376.992 cu. ft.

6. Diameter of the base 6 inches, altitude 5 feet, to find the volume. Ans. 1,696.464 cu. in.

7. Radius of the base 6 feet, altitude 20 feet, to find the volume. Ans. 2,261.952 cu. ft.

8. Radius of the base 10 feet, altitude 30 feet, to find the volume. Ans. 9,424.8 cu. ft.

9. Circumference of the base 31.416 feet, altitude 40 feet, to find the volume. Ans. 3,141.6 cu. ft.

10. Circumference of the base 80 feet, altitude 50 feet, to find the volume. Ans. 25,464+ cu. ft.

11. Find the cost of a stick of timber 20 inches square and 30 feet long, at 25 cents a cubic foot. Ans. $20.83\frac{1}{3}$.

12. Find the number of cubic feet in a log 30 feet long and 24 inches in diameter at each end.
Ans. 94.248 cu. ft.

13. Find the capacity in gallons of a cylindrical cistern measuring 8 feet across and 20 feet deep.
Ans. 7,520.256 gals.

14. Find the value of a piece of moulding, the end of which is a triangle whose sides are respectively 3, 4, and 5 inches, and length 24 feet, at 50 cents per cubic foot.
Ans. 50 cts.

15. What is the comparative weight of two poles of the same material, one being 3 inches in diameter and 10 feet long, and the other 6 inches in diameter and 20 feet long? Ans. latter = 8 times former.

16. Find the volume of a prism whose base contains $8\frac{1}{6}$ square feet, and the square of whose height equals six times the number of square feet in the base.
Ans. $57\frac{1}{6}$ cu. ft.

17. A lime kiln in the form of a prism is 12 feet square and 24 feet high; through the center of this, from top to bottom, there is a cylindrical opening 6 feet in diameter; required the number of perches of masonry in the kiln. Ans. 112.218+P.

PYRAMIDS AND CONES

DEFINITIONS

86. A **Pyramid** is a solid having for its base a polygon and for its sides plane triangles, all terminating at one point, called the vertex.

When the base is regular the pyramid is regular; otherwise irregular.

87. A **Cone** is a solid whose base is a circle, and whose lateral curved surface tapers uniformly to a point, called the vertex.

88. The **Altitude** of a pyramid or of a cone is the perpendicular distance from the vertex to the plane of the base.

89. The **Axis** of a pyramid or of a cone is the straight line that joins the vertex with the center of the base.

90. A **Right Pyramid** or a **Right Cone** is one whose axis is perpendicular to the base of the pyramid or cone. The base of the pyramid is a regular polygon.

91. An **Oblique Pyramid** or an **Oblique Cone** is one whose axis is oblique to the base of the pyramid or cone.

92. The **Slant Height** of a pyramid is the perpendicular distance from its vertex to either of the sides of the base.

93. The **Slant Height** of a cone is a straight line drawn from the vertex to the circumference of the base.

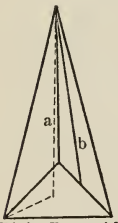
94. A **Right Cross Section** of a pyramid or of a cone is a section made by a plane perpendicular to the axis.

95. A **Frustum** of a pyramid or of a cone is that part which remains after cutting off the top by a plane parallel to the base.

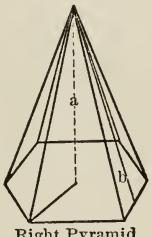
96. The **Altitude of a Frustum** of a pyramid or of a cone is the perpendicular distance between the bases.

97. The **Slant Height of the Frustum** of a right pyramid or of a right cone is the shortest distance between the perimeters of the bases.

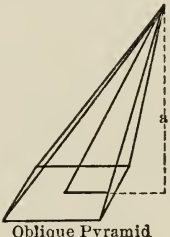
98. A pyramid is triangular, quadrangular, pentagonal, hexagonal, etc., according as its base is a triangle, quadrilateral, pentagon, hexagon, and so on.



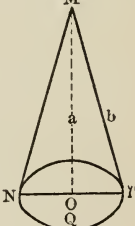
Right Pyramid (Triangular) (*Fig.* 1)



Right Pyramid (Hexagonal) (*Fig.* 2)



Oblique Pyramid (Quadrangular) (*Fig.* 3)



Right Cone (*Fig.* 4)

In Fig. 4, if the right-angled triangle MON be revolved around MO as an axis, it will generate the cone NQP—M.

In Fig. 7, if the trapezoid MNOP be revolved around MO as an axis, it will generate the frustum of the cone QRP—N.

In Figs. 1, 2, 3, 4, 5, 6, and 7, the line representing the altitude is marked a.

In Figs. 1, 2, 4, 6, and 7, the line representing the slant height is marked b.

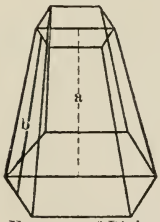
All the lateral faces of a right pyramid are equal triangles; the sum of the bases of these triangles is equal to the perimeter of the base of the right pyramid.

The altitude of each lateral triangle is equal to the slant height of the pyramid; moreover, a cone may be regarded as a pyramid of an infinite number of sides. The reason for the following rule is, therefore, obvious:

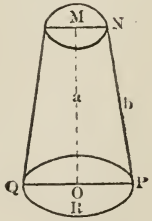
Rule I. To find the *convex* surface of a right pyramid or of a right cone: *Multiply the perimeter or circumference of the base by one-half the slant height.*

To find the *entire* surface, add to the convex surface the area of the base. The *lateral* surface of any pyramid may be found by adding together the areas of the lateral faces.

Oblique Cone
(*Fig.* 5)



Frustum of Right Pyramid
(*Fig.* 6)



Frustum of Right Cone
(*Fig.* 7)

99. Each lateral face of the frustum of a right pyramid is a trapezoid. The slant height of the frustum is equal to the perpendicular distance between the parallel sides of the trapezoid. The perimeter of the lower base of the frustum is equal to the sum of the lower bases of the trapezoids forming the convex surface, and the perimeter of the upper base of the frustum is equal to the sum of the upper bases of the same trapezoids. Now, the frustum of a cone may be regarded as the

frustum of a pyramid of an infinite number of sides. The rule for finding the area of a trapezoid is, therefore, applicable.

Rule II. To find the *convex* surface of the frustum of a right pyramid or cone: *Multiply the sum of the perimeters or circumferences of the two bases by one-half the slant height.*

For the *entire* surface, add to the convex surface the area of both ends or bases.

Rule III. To find the volume of a pyramid, cone, frustum of a pyramid, or frustum of a cone: *Multiply the sum of both end areas and four times the area of a section half way between them, by one-sixth the altitude.* $\frac{a}{6}(B + b + 4m)$, or,

To find the volume of a pyramid or cone: *Multiply the area of the base by one-third the altitude.*

To find the volume of the frustum of a pyramid or of a cone: *To the sum of the areas of both bases add the square root of their product, and multiply this sum by one-third the altitude.*

SURFACES OF PYRAMIDS AND CONES AND THEIR FRUSTUMS

PROBLEMS

1. What is the lateral surface of a quadrangular pyramid, the sides of whose base are each 5 feet and slant height 30 feet? Ans. 300 sq. ft.

2. What is the entire surface of the pyramid mentioned in Problem 1? Ans. 325 sq. ft.

3. What is the lateral surface of a hexagonal pyramid, the sides of whose base are each 8 feet and slant height 40 feet? Ans. 960 sq. ft.

4. Find the cost of painting an octagonal church-spire at 22 cents per square yard, the sides of whose base are each 4 feet and slant height 60 feet. Ans. $\$23.46\frac{2}{3}$.

5. Find the convex surface of a cone, the diameter of whose base is 10 feet and slant height 20 feet.
Ans 314.16 sq. ft.

6. Find the entire surface of the cone mentioned in Problem 5. Ans. 392.7 sq. ft.

7. How many square yards of canvas will it take to make a conical tent whose center-pole is 8 feet high and the diameter of the base 12 feet? Ans. 20.944 sq. yd.

8. Required the lateral surface of the frustum of a square pyramid whose slant height is 40 feet, the side of the lower base 20 feet, and upper base 16 feet.
Ans. 2,880 sq. ft.

9. What is the entire surface of the frustum mentioned in Problem 8? Ans. 3,536 sq. ft.

10. What is the lateral surface of the frustum of a hexagonal pyramid whose slant height is 20 feet, the sides of the lower base each 6 feet, and upper base 4 feet?
Ans. 600 sq. ft.

11. Find the convex surface of the frustum of a cone whose slant height is 30 feet, the circumference of the lower base 40 feet, and of the upper base 30 feet.
Ans. 1,050 sq. ft.

12. Find the entire surface of the frustum mentioned in Problem 11. Ans. 1,248.943+ sq. ft.

13. What is the convex surface of the frustum of a square pyramid whose slant height is 10 inches, the side of the upper base being 18 inches and of the lower base 20 inches? Ans. 760 sq. in.

VOLUMES OF PYRAMIDS AND CONES

PROBLEMS

In a pyramid, given:

1. The side of a square base 4 feet, altitude 30 feet, to find the volume. Ans. 160 cu. ft.

2. The side of a square base 8 inches, altitude 21 feet, to find the volume. Ans. 3 cu. ft., 192 cu. in.

3. The area of the square base 16 square yards, altitude 30 yards, to find the volume. Ans. 160 cu. yd.

4. The sides of a rectangular base 60×40 feet, altitude 90 feet, to find the volume. Ans. 72,000 cu. ft.

5. Find the volume of a square pyramid whose altitude is 60 feet, and the side of whose base is 9 feet. Ans. 1,620 cu. ft.

6. The base of a right triangular pyramid is an equilateral triangle, each side of which is 12 feet and the altitude 18 feet; find the cubic contents. Ans. 374.1228+ cu. ft.

7. The altitude of a pyramid is 8 feet, and its base is a rectangle 3 feet by 2; find the volume. Ans. 16 cu. ft.

8. A certain pyramid has an altitude equal to twice the diagonal of its base, which is a rectangle 60 by 80 feet; find its volume. Ans. 320,000 cu. ft.

9. Find the volume of a pyramid whose base is a rectangle 40 feet by 30, and the length of the edges which meet at the vertex 65 feet. Ans. 24,000 cu. ft.

10. A granite monument consists of a pedestal 24 inches square and 4 feet high, on which stands a pyramid 18 inches square and 9 feet high; what did it cost at $15 per cubic foot? Ans. $341.25.

In a cone, given:

11. The diameter of the base 6 feet, altitude 18 feet, to find the volume. Ans. 169.6464 cu. ft.

12. The diameter of the base 2 feet, altitude 12 feet, to find the volume. Ans. 12.5664 cu. ft.

13. The radius of the base 10 inches, altitude 15 inches, to find the volume. Ans. 1,570.8 cu. in.

14. The circumference of the base 62.832 feet, altitude 30 feet, to find the volume. Ans. 3,141.6 cu. ft.

15. Find the solid contents of a cone whose altitude is 48 feet and the diameter of whose base is 5 feet. Ans. 314.16 cu. ft.

16. How many tons of hay in a conical stack 30 feet high and 40 feet across the base, there being 500 cubic feet to the ton? Ans. 25.1328 tons.

17. Mr. Johnston's crop of wheat piled on his barn floor formed a heap in the form of a cone 10 feet high and 62.832 feet in circumference; what was it worth at $1.25 per bushel? Ans. $1,051.865.

18. Mr. Brown's granary sprang a leak 2 feet above the level of his barn floor; the clover seed that ran out of this opening formed a heap in the shape of a half-cone 2 feet high, and extended 2 feet at the bottom from the perpendicular partition between the barn floor and the granary; how many bushels leaked out? Ans. 3.3659+ bu.

19. The circumference of the base of a cone is 15.708 feet, and the altitude is equal to the diameter of the base; find its volume. Ans. 32.725 cu. ft.

20. Compare the volumes of a cone and a cylinder, the altitude of each being 12 feet and the radius of the base of each 4 feet. Ans. Cone $= \frac{1}{3}$ vol. cyl.

VOLUMES OF FRUSTUMS OF PYRAMIDS AND CONES

PROBLEMS

Given, in the frustum of a pyramid:

1. Upper side of square base, 10 feet; lower side, 12 feet; altitude, 20 feet; to find the volume.
Ans. $2,426\frac{2}{3}$ cu. ft.

2. Upper sides of rectangular base, 12 and 16 feet; lower sides, 18 and 24 feet; altitude, 30 feet; to find the volume. Ans. 9,120 cu. ft.

3. Upper sides of triangular base, each 6 feet; lower sides of triangular base, each 8 feet; altitude, 18 feet; to find the volume. Ans. 384.50+ cu. ft.

4. Find the volume of the frustum of a square pyramid whose altitude is 30 feet, each side of the lower base 24 feet, and of the upper base 18 feet.
Ans. 13,320 cu. ft.

5. Find the volume in cubic feet of a stick of timber 36 feet long, the larger end being 18 inches square and the smaller end 15 inches square. Ans. $68\frac{1}{4}$ cu. ft.

6. How many cubic feet in the upright of a crane 30 feet high, the lower end being 2 feet square and the upper end 18 inches square? Ans. $92\frac{1}{2}$ cu. ft.

7. The lower base of the frustum of a pyramid is a rectangle 20 by 40 inches; the upper base, a rectangle 15 by 30 inches, and the altitude 12 feet; what is the volume? Ans. 51 cu. ft. 672 cu. in.

8. The smaller base of the frustum of a pyramid is a rectangle 3 by 4 feet; the longer side of the lower base is equal to the diagonal of the upper base, and the shorter side of the lower base bears the same ratio to the longer side as the shorter side of the upper base

bears to its longer side; the altitude is equal to the sum of the perimeters of both bases. Required the volume. Ans. 480.375 cu. ft.

9. What are the solid contents of the frustum of a cone whose upper base is 10 feet in diameter, lower base 16 feet, and altitude 18 feet? Ans. 2,431.5984 cu. ft.

10. Find the number of cubic feet in a log 42 feet long, the diameter of the larger end being 3 feet and of the smaller end 2 feet. Ans. 208.9164 cu. ft.

11. A telephone pole 48 feet long is 5 feet in circumference at the bottom and 3 feet at the top; required the contents. Ans. 62.387+ cu. ft.

12. Find the capacity in gallons of a pail 18 inches deep, measuring 14 inches across the top and 12 inches across the bottom. Ans. 10.363+ gal.

13. A tub 2 feet deep is 26 inches in diameter at the bottom and 30 inches at the top; how many pails of water the size of the one described in Problem 12 will be required to fill the tub? Ans. 6.1+ pails.

14. Mr. Long's cistern is 7 feet deep, the bottom being 6 feet in diameter and the top 4 feet; how many barrels of water will it hold? Ans. 33.07+ bbl.

15. How many quarts of wine will be required to fill 2 dozen tumblers, each 2 inches across the bottom, 3 inches across the top, and 4 inches deep? Ans. 8.26+ qt.

16. A butcher's pickle-stand is 5 feet deep, 6 feet in diameter at the bottom, and 4 feet at the top; how many gallons will it contain? Ans. 744.192 gal.

17. A farmer had a conical stack of hay 314.16 feet in circumference at the bottom and 36 feet high. A storm blew away the top so that the remainder was found

to be only 18 feet high; how many cubic feet remained in the stack? How many cubic feet blew away?

Ans. Remained, 82,467 cu. ft.; blew away, 11,781 cu. ft.

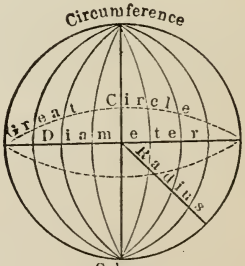
18. At $15 per cubic foot, find the difference in the cost of two granite monuments, one of which is the frustum of a square pyramid 3 feet square at the top, 4 feet square at the bottom, and 30 feet high; the other is the frustum of a cone 30 feet high, the diameter of the upper base being 3 feet and of the lower base 4 feet.

Ans. $1,191.03.

THE SPHERE

DEFINITIONS

100. A **Sphere** is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center. It may be generated by the revolution of a semi-circle about its diameter as an axis.



Sphere
Fig. 1

101. A **Diameter** of a sphere is a straight line passing through the center of the sphere, and terminated at both ends by the surface.

102. A **Radius** of a sphere is a straight line drawn from the center to any point in the surface. It is half the diameter.

103. A **Zone** is the portion of the surface of a sphere included between two parallel planes.

104. A **Spherical Segment** is the solid portion of a sphere included between two parallel planes.

105. A **Spherical Sector** is a volume generated by the revolution of a circular sector about the diameter.

106. A **Great Circle** is a section made by any plane passing through the center of a sphere.

Rule I. To find the surface of a sphere: *Multiply the circumference of the sphere by the diameter.* Or, $S = \pi D^2$.

Rule II. *Multiply the square of the radius by four times 3.1416.* Or, $S = 4\pi R^2$.

Rule III. To find the volume of a sphere: *Multiply the sum of both end areas and four times the area of a section half way between them, by one-sixth the altitude.* $\frac{a}{6}(B + b + 4m)$

Rule IV. *Multiply the surface by one-sixth the diameter, or one-third the radius.*

Rule V. *Multiply the cube of the diameter by one-sixth of 3.1416, or by .5236.* Or, $V = \frac{\pi D^3}{6}$.

107. The sphere may be regarded as a polyedron consisting of an infinite number of pyramids whose combined bases form the surface of the sphere and whose equal altitudes are equal to the radius of the sphere. Since the volume of each pyramid is equal to its base multiplied by one-third of its altitude, and since the whole is equal to the sum of all its parts, Rule IV is readily derived.

The cylinder, the cone, and the sphere are the three round bodies of Geometry. There is a fixed relation existing between these three solids, both as to their surfaces and their volumes. See Problems 22 and 23, Art. 114.

SURFACES OF THE SPHERE

PROBLEMS

In a sphere, given:

1. The diameter 8 inches, to find the surface.
Ans. 201.06+ sq. in.

2. The diameter 18 inches, to find the surface.
Ans. 1,017.9— sq. in.

3. The diameter 10 inches, to find the surface.
Ans. 314.16 sq. in.

4. The radius 3 inches, to find the surface.
Ans. 113.1— sq. in.

5. The radius 8 inches, to find the surface.
Ans. 804.25— sq. in.

6. The radius 10 inches, to find the surface.
Ans. 1,256.6+ sq. in.

7. The circumference 62.832 inches, to find the surface. Ans. 1,256.6+ sq. in.

8. The circumference 15.708 inches, to find the surface. Ans. 78.54 sq. in.

9. The circumference 251.328 inches, to find the surface. Ans. 20,106.24 sq. in.

10. How many square inches of tissue paper will be required to wrap 20 oranges each 2 inches in diameter, provided no allowance is made for waste?
Ans. 251.328 sq. in.

11. How many square inches of pig skin in a spherical football 9 inches in diameter? Ans. 254.47— sq. in.

12. How many square inches of bronze will it take to cover the three balls used as a sign of a pawnbroker, if each ball is 4 inches in diameter?
Ans. 150.796+ sq. in.

VOLUMES OF THE SPHERE

PROBLEMS

In a sphere, given :

1. The diameter 2 feet, to find the volume.
Ans. 4.1888 cu. ft.

2. The diameter 5 inches, to find the volume.
Ans. 65.45 cu. in.

3. The radius 23 inches, to find the volume.
Ans. 50,965.1296 cu. in.

4. The circumference 37.6992 feet, to find the volume.
Ans. 904.7808 cu. ft.

5. How much will 16 croquet balls each 4 inches in diameter weigh, if a cubic inch of the wood of which they are made weighs $\frac{1}{2}$ ounce? Ans. 16 lbs. 12.08+ oz.

6. If the croquet balls mentioned in Problem 5 were placed in a rectangular box 16 inches long, 8 inches deep, and 8 inches wide, how many quarts of sand (dry measure) would be required to fill all the remaining space and make the box level full? Ans. 7.259+ qt.

7. Admiral Dewey at the battle of Manila used a number of 8-inch guns ; what would be the weight of a solid spherical ball made to fit these guns, if a cubic foot of iron weighs 450 pounds? Ans. 69.81251b.

8. A sphere is 20 inches in diameter ; what is the volume of an inscribed cube? Ans. 1,539.598+ cu. in.

9. The edge of a cube is 20 inches ; what is the volume of an inscribed sphere? Ans. 4,188.8 cu. in.

10. The outside diameter of a hollow globe of uniform thickness is 16 inches and the inside diameter 14 inches ; how many cubic inches of metal in the shell?
Ans. 707.9072 cu. in.

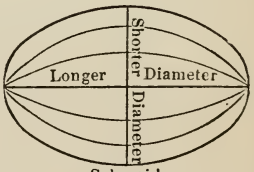
11. The main battery of the battleship Oregon has four 13-inch, eight 8-inch, and four 6-inch guns; if solid spherical balls were used instead of projectiles, what weight of metal would be required to load these once round, there being 450 pounds of the metal used to the cubic foot? Ans. 1,874.59+lb.

12. A hollow sphere having an inside diameter of 12 inches was filled with water; if a solid ball 3 inches in diameter, a solid cylinder having a base 2 inches in diameter and 4 inches high, and a solid square pyramid whose base is 3 inches on a side and altitude 6 inches, were placed inside the sphere so as to displace part of the water, how many cubic inches of water would remain? Ans. 860.0772 cu. in.

THE SPHEROID

DEFINITIONS

108. A **Spheroid** is a volume formed by the revolution of an ellipse about one of its axes. A revolution about the longer axis forms a *prolate* spheroid; and about the shorter axis, an *oblate* spheroid.



Spheroid or Ellipsoid
*Fig.*1

Rule I. To find the volume of a spheroid: *Multiply the sum of both end areas and four times the area of a section half way between them, by one-sixth the altitude.* $\frac{a}{6}(B + b + 4m)$.

NOTE.— For the spheroid each end area is Zero, and either axis may be used as the altitude, according as the area of the middle section is found for the prolate or oblate spheroid.

Rule II. *Multiply the square of the revolving axis by the fixed axis and this product by .5236.*

VOLUMES OF THE SPHEROID

PROBLEMS

1. What is the volume of a spheroid whose longer diameter is 24 inches and shorter diameter 20 inches?

Ans. 5026.56 cu. in.

2. Find the volume of a stone in the shape of a spheroid whose longer diameter is 4 feet and shorter diameter 2 feet. Ans. 8.3776 cu. ft.

3. A watermelon is shaped like a spheroid; what are its contents, if its longer diameter is 18 inches and shorter diameter 12 inches? Ans. 1,357.1712 cu. in.

4. How many cubic inches of air in a spheroidal football whose longer inside diameter is 14 inches and shorter diameter 9 inches? Ans. 593.7624 cu. in.

5. Which has the greater volume and how much, a sphere 12 inches in diameter, or a spheroid whose longer axis is 8 inches and shorter axis 4 inches?

Ans. The sphere, 837.76 cu. in. greater.

6. Which has the greater volume and how much, a spheroid whose longer diameter is 30 inches and shorter diameter 20 inches, or the frustum of a square pyramid whose lower base is 10 inches square, upper base 8 inches square, and whose altitude is 30 inches?

Ans. The spheroid, 3843.2 cu. in. greater.

7. A sphere whose radius is 8 inches and a spheroid having a longer diameter of 18 inches and shorter diameter of 12 inches, were placed in a tub in the form of a frustum of a cone; the lower diameter of the tub was 2 feet, the upper diameter 26 inches, and depth 18 inches; how many gallons of water would have been required to fill the remaining space? Ans. 23.11+ gal.

CIRCULAR RINGS

DEFINITIONS

109. A **Circular Ring** is formed by bending a cylinder or a bar until the two ends meet.

Fig.3

Circular Rings

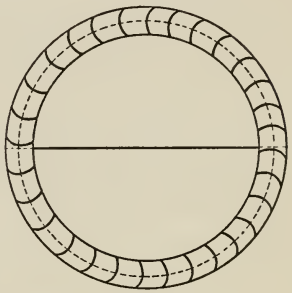


Fig.1

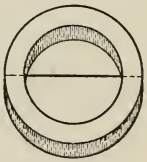


Fig.2

It will be observed by examining Figs. 1 and 3 that if the ring is cut at any place and made straight the same volume will be changed into the form of a cylinder. The altitude of the cylinder will equal one-half the sum of the inner and outer circumferences, and either base of the cylinder will be equal to any cross section of the ring. The rules for finding both the surface and the volume, therefore, are practically the same as those for finding the convex surface and volume of a cylinder.

Rule I. To find the surface of a circular ring: *Multiply the perimeter of a cross section of the ring, or its girt, by one-half the sum of the inner and outer circumferences.*

Rule II. To find the volume of a circular ring: *Multiply both end areas (conceiving the ring to be made straight) and four times the area of a section half way between them, by one-sixth the altitude.* $\frac{a}{6}(B + b + 4m)$.

Rule III. *Multiply the area of a cross section of the ring by one-half the sum of the inner and outer circumferences.*

SURFACES OF CIRCULAR RINGS

PROBLEMS

1. What is the surface of a pneumatic bicycle tire whose outside diameter is 28 inches and thickness $1\frac{1}{2}$ inches? Ans. 392.3186+ sq. in.

2. A hose whose outside diameter is 2 inches was placed in the form of a ring; the outer circumference measured 62.832 feet; required the surface of the hose. Ans. 32.6246+sq.ft.

3. The felloes of a heavy lumber wagon were made 2 inches deep and 4 inches wide; the outside diameter of each hind wheel, inside the tire, was 4 feet 4 inches; how many square feet of surface are there to paint on each felloe of the hind wheels, provided the four sides are painted before putting on the tire, and no allowance is made for shaving off the corners on the inside? Ans. 13.09 sq. ft.

4. A semi-circular white oak arch of uniform thickness and 3 inches in diameter, was made to span the main street of a town; the distance on the inside between the ends of the poles where buried in the ground was 30 feet; required the number of square feet of bunting needed to cover the arch. Ans. 37.3196+ sq. ft.

VOLUMES OF CIRCULAR RINGS

5. What is the volume of a solid iron ring of the same dimensions as the pneumatic bicycle tire in Problem 1? Ans. 147.1194+ cu. in.

6. Find the solid contents of a grape vine of exactly the same shape and dimensions as the hose in Problem 2. Ans. 1.3593+ cu. ft.

7. Find the number of cubic inches in each felloe described in Problem 3. Ans. 1,256.64 cu. in.

8. What is the weight of the white oak arch in Problem 4, if a cubic foot of wood weighs 51 pounds?
Ans. 118.956+ lb.

THE WEDGE

DEFINITIONS

110. A **Wedge** is a volume of five sides, having a rectangular base, two rectangular or trapezoidal sides meeting in an edge, and two triangular ends.

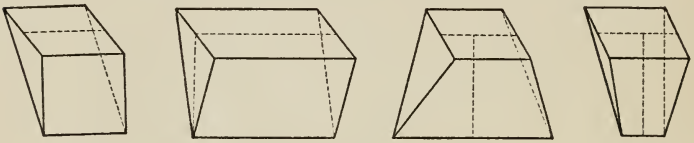


Fig.1 *Fig.2* Wedges *Fig.3* *Fig.4*

Rule I. To find the surface of a wedge: *Find the sum of the areas of the five faces.*

Rule II. To find the volume of a wedge: *Multiply the sum of the two end areas and four times the area of a section half way between them, by one-sixth the altitude.* $\frac{a}{6}(B + b + 4m)$.

In the wedge, the end area where the sides meet is Zero.

SURFACES OF THE WEDGE

PROBLEMS

To find the surface of a wedge, given:

1. Base of wedge, a rectangle 6 x 4 inches; sides, equal rectangles; ends, equal isosceles triangles; altitude of

wedge, 12 inches; altitude of rectangle, 12.1655 inches. (Fig. 2.) Ans. 217.986 sq. in.

NOTE.—The altitudes of the wedges are given in these surface problems; from these the pupil may be required to find the altitudes of the ends and sides, when time will permit.

2. Base of wedge, a square 4 x 4 inches; sides, equal trapezoids 4 inches along the base and 8 inches along sharp edge; ends, two equal isosceles triangles; altitude of wedge, 6 inches; altitude of each trapezoid, 6.3246 inches; altitude of each triangle, 6.3246 inches. (Fig. 3.) Ans. 117.1936 sq. in.

3. Base of wedge, a square 6 x 6 inches; sides, 2 equal trapezoids whose edges next the base are each 6 inches and sharp edge 4 inches; ends, two equal isosceles triangles; altitude of wedge, 8 inches; altitude of trapezoids, 8.544 inches; altitude of triangles, 8.0623 inches. (Fig. 4.) Ans. 169.8138 sq. in.

VOLUMES OF THE WEDGE

To find the volume of a wedge, given:

4. The base of the wedge, a rectangle 16 x 12 inches; the sharp edge, 16 inches; triangular ends both perpendicular to the base of the wedge; altitude of the base, 21 inches. (Fig. 2.) Ans. 2,016 cu. in.

5. The base of the wedge, a square 12 x 12 inches; the sharp edge, 20 inches; the altitude of the wedge, 30 inches. (Fig. 3.) Ans. 2,640 cu. in.

6. The base of the wedge, a square 8 x 8 inches; sharp edge, 6 inches; altitude of wedge, 12 inches. (Fig. 4.) Ans. 352 cu. in.

7. A plane was passed from the upper edge of a cube, which edge is 6 inches, to the lower parallel edge on the opposite side of the cube; required the volume of either wedge thus made. (Fig. 1.) Ans. 108 cu. in.

SIMILAR SOLIDS

DEFINITIONS

111. Similar Solids are such as have the same form, and differ from each other only in size; as, cubes, spheres, etc.

112. A **Dimension** of a solid is a radius, diameter, circumference, height, length, breadth, etc.

113. The following principles of similar solids are derived from geometry:

PRINCIPLE I. *Similar solids are to each other as the cubes of their like dimensions.*

PRINCIPLE II. *Like dimensions of similar solids are to each other as the cube roots of those solids.*

PROBLEMS

1. If the volume of a cube 4 inches on each side is 64 cubic inches, what is the volume of one 5 inches on each side?

OPERATION.—$4^3 : 5^3 :: 64$ cu. in. : X. $X = 125$ cu. in., Ans.

2. How many baseballs each 2 inches in diameter are equal in volume to one large ball 1 foot in diameter? Ans. 216.

3. If a conical stack of wheat in the sheaf 20 feet high contains 400 bushels, how many bushels should a similar stack 30 feet high contain? Ans. 1,350.

4. A farmer can load on wagons the contents of a cubical bin 10 feet on a side in half a day; how long, at the same rate, will it take him to load the contents of a cubical bin 20 feet on a side? Ans. 4 days.

5. If a colt $13\frac{1}{2}$ hands high weighs 1,000 pounds, how much should he weigh when he becomes a horse 15 hands high? Ans. 1,371.7+ lbs.

6. If a barrel whose bung-diameter is 38 inches, when filled contains 64 gallons, what would be the bung-diameter of a similar cask that holds but 27 gallons?

Ans. $28\frac{1}{2}$ in.

7. If the volume of a pyramid whose altitude is 25 feet is 729 cubic feet, what is the altitude of a similar pyramid whose volume is 9,261 cubic feet?

Ans. $58\frac{1}{3}$ ft.

8. If a cylindrical tank 2 feet in diameter contains 4 barrels of water, what is the capacity of a similar tank 4 feet in diameter? Ans. 32 bbl.

9. John is 4 feet tall and weighs 64 pounds; what should be his height when he weighs 216 pounds, provided he develops proportionately? Ans. 6 ft.

10. If it requires $31\frac{1}{4}$ tons of ice to fill a cubical ice house whose edge is 10 feet, how many tons will be required to fill one whose edge is 20 feet?

Ans. 250 tons.

11. The diameters of three globes are respectively $\frac{1}{3}$, $\frac{4}{9}$, and $\frac{5}{9}$ of an inch; what is the diameter of a globe equal to the volume of the three? Ans. $\frac{2}{3}$ in.

12. The patriotic citizens of a certain city wished to raise \$8,000 for the purpose of erecting a monument to the heroes who went down with the "Maine" in Havana harbor. The monument was to be the frustum of a square pyramid 50 feet high. Patriotism ran so high, however, that the sum of \$15,625 was raised; how high a monument did the latter sum erect, provided the original design of the monument was carried out?

Ans. $62\frac{1}{2}$ ft.

MISCELLANEOUS PROBLEMS

114. Review.

1. Required the distance between an upper corner and an opposite lower corner of a church 96 feet long, 72 feet wide, and 25 feet high. Ans. 122.576+ ft.

2. How many square yards of plastering were required to cover the walls and ceiling of the church described in the preceding problem, deducting 80 square yards for openings? Ans. $1{,}621\frac{1}{3}$ sq. yds.

3. John's bicycle wheels are each 28 inches in diameter; how many times will they revolve in going 8,796.48 inches? Ans. 100 times.

4. A gas receiver in a certain city is a cylinder 40 feet in diameter; on an average during the year each morning it is 6 feet deeper in the water than the evening before; how many cubic feet of gas are consumed each night?
Ans. 7,539.84 cu. ft.

5. The diameter of a cylindrical vessel filled with water is 10 inches. An immersed stone displaces 2 inches of the depth of the water. How many cubic inches are there in the stone? Ans. 157.08 cu. in.

6. A teamster hauled from a mill 40 quadrangular posts, each 6 by 8 inches and 12 feet long. How many triangular posts, the base of each of which is a triangle measuring 6, 8, and 10 inches on a side respectively, and the same length, could he have hauled, provided the loads were equal in weight? Ans. 80 posts.

7. A hall 60 by 120 feet has a roof in the form of a wedge. Both ends are equal triangles and both sides equal trapezoids; the two sides meet at the ridge, which

is 80 feet long and 30 feet above the top of the walls. Required the length of a rafter extending from a corner of the building to the ridge? Ans. 46.904+ ft.

8. Required the number of square feet in the entire roof as described in the preceding problem.

Ans. 10,648.58+ sq.ft.

9. How many cubic feet of air are enclosed by the roof and attic floor of the hall described in Problem 7?

Ans. 96,000 cu. ft.

10. In cutting the grass on a lawn 60 feet wide and containing 4,800 square feet, how many times must a lawn-mower 18 inches wide be drawn over it lengthwise so that the whole may be cut? Ans. 40 times.

11. Mr. Clark, wishing to give his three boys practical lessons in both industry and foresight, made the following proposition: "You may each have sufficient lumber to build 60 rods of fence, with which you may enclose as much land as you can; all that you raise on the land thus enclosed may be your own; I should like to see you make as much of my offer as you possibly can." The oldest enclosed a circular piece; the second, a square piece, and the youngest, a rectangular piece twice as long as it was broad; how many square rods did each have in his field?

Ans. Oldest, 286.478+ sq. rds.; second, 225 sq. rds.; youngest, 200 sq. rds.

12. Find the length of a hand-rail for a flight of stairs having 20 steps, each step $7\frac{1}{2}$ inches high and 9 inches wide. Ans. 19.525+ ft.

13. In Brandon Park there is a regular octagonal stand. Surrounding this stand there is a porch of uniform width. The perimeter of the stand is $61\frac{1}{3}$ feet; the

perimeter of the outer edge of the porch is $94\frac{2}{3}$ feet; the perpendicular distance between the stand and the outer edge of the porch, or the width of the porch, is 5 feet 2 inches. How many square feet of flooring in the porch floor? Ans. 403 sq. ft.

14. The frame and handle of a corn-popper is made of a continuous piece of wire. The handle is 12 inches long, and the frame upon which the wire sieve is fastened is a rectangle 8 by 4 inches; how long a wire was required to make this frame and handle, making no allowance for the thickness of the wire? What is the combined length of the handle and frame? Ans. 36 in.; 20 in.

15. A lamp having a paper shade is placed on a stand in the center of a room 18 by 20 feet; the shade casts on the ceiling a shadow in the form of a circular ring extending the entire width of the ceiling and uniformly 5 feet wide. How many square feet of the ceiling are not shaded? Ans. 155.796 sq. ft.

16. Mr. Jones gave each of his two sons half a dollar to spend. The younger invested his in 5-cent tickets for a merry-go-round, each ticket allowing him to ride on the merry-go-round, 80 feet in circumference, 10 times. The older bought a railroad ticket at 3 cents a mile. How many miles more did the older ride than the younger? Ans. 15.15 mi.

17. The width of Mr. F's buggy from center of tire to center of tire is 4 feet 9 inches; the diameter of the fore wheel is 43 inches and of the hind wheel 46 inches. This buggy is drawn round a circular race-course so that the outside fore wheel leaves a track exactly one mile long. The buggy is so coupled that the track made by the outside hind wheel, from center of track to center of track, is at all places one inch distant from the track made by

the outside fore wheel. How many revolutions does each wheel of the buggy make in going once round the track?

Ans. Outside fore wheel, 469.02+ times; outside hind wheel, 438.39+ times; inside front wheel, 466.37+ times; inside hind wheel, 435.91+ times.

18. A mechanic converted a cylindrical piece of iron 2 inches in diameter and 2 feet long into a wire $\frac{1}{4}$ of an inch in diameter; what was the length of the wire? Ans. 128 ft.

19. Lamar Township purchased tiling for drainage. Each section of the tiling is a hollow cylinder 28 inches high, the outer diameter being 36 inches and the inner diameter 31 inches; how many cubic feet of material in each section, no allowance being made for the flange?

Ans. 4.2633+ cu. ft.

NOTE.— The pupil should work the above problem by using the area of the circular ring as a base, and also by finding the volume of two cylinders and taking their difference.

20. Each side of the lower base of the frustum of a square pyramid is 4 feet, of the upper base 3 feet, and slant-height 20 feet. The perimeter of the lower base of the frustum of a cone is 16 feet, of the upper base 12 feet, and slant-height 20 feet. How do the lateral surfaces compare? How do the entire surfaces compare?

Ans. 6.83+ sq. ft.; frus. cone greater.

21. Mr. Johnson's heater has a cylindrical magazine 2 feet deep and 10 inches in diameter; how many shovelfuls of coal will be required to fill it, if 1 shovelful weighs 8 pounds, and coal weighs 60 pounds to the cubic foot? Ans. 8.18125 shovelfuls.

22. Compare the volumes of a cylinder, a sphere, and a cone, whose dimensions are as follows: Cylinder— diameter of base and altitude, each 20 inches; sphere—

diameter, 20 inches; cone—diameter of base and altitude, each 20 inches.

Ans. 2 vol. cyl. = 3 vol. sphere; 1 vol. cyl. = 3 vol. cone; 1 vol. sphere = 2 vol. cone.

23. Compare the entire surfaces of the cylinder and sphere given in the preceding problem.

Ans. 2 sur. cyl. = 3 sur. sphere.

24. A globular piece of wax 6 inches in diameter, a conical piece whose base is 6 inches and altitude 12 inches, and a cylindrical piece whose diameter is 6 inches and altitude 12 inches, were melted and then made into one cubical piece; required the edge of the cube. Ans. 8.26+ in.

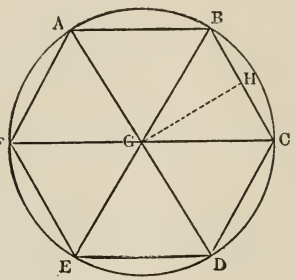
25. A farmer placed on the level ground a heap of apples in the form of a cone whose diameter was 6 feet and altitude 6 feet; this he covered with earth of uniform thickness, making when complete a cone similar to the heap of apples; the diameter of the larger cone thus formed was 8 feet. How many cubic feet of earth in the covering? Ans. 77.4928 cu. ft.

26. A granite monument consists of three pieces—a rectangular solid, a frustum of a square pyramid, and a globe; the rectangular solid is 6 feet square and 4 feet high; the lower base of the frustum of the pyramid is 5 feet square, the upper base 3 feet square, and altitude 24 feet; the globe is 2 feet in diameter. Required the weight of the entire monument, if a cubic foot of granite weighs 168 pounds? Ans. 45.3758+ tons.

27. A hollow squash shaped like an ellipsoid has an outside longer diameter of 12 inches and outside shorter diameter of 8 inches; the average thickness of the substance is 2 inches. How many cubic inches of material in the squash? Ans. 335.104 cu. in.

28. The lower joint of a stove pipe is 2 feet long; the upper end is a circle 6 inches in diameter, and the lower end an ellipse whose transverse axis is 7 inches and conjugate axis 5 inches. How many cubic inches of sand would be required to fill it? Ans. 672.3024 cu. in.

29. The accompanying diagram represents a regular hexagon inscribed in a circle, the radius of which is 6 inches. Each side of a regular inscribed hexagon is equal to the radius of the circumscribing circle. Hence, GB = 6 inches, and any side of the hexagon, as BC = 6 inches.



(*a*) Find the length of GH. Ans. 5.19615+ in.

(*b*) Find in two ways the area of the triangle GBC. Ans. 15.58845 sq. in.

(*c*) Find in two ways the area of the rhombus ABCG. Ans. 31.1769 sq. in.

(*d*) Find in two ways the area of the trapezoid ABCD. Ans. 46.76535 sq. in.

(*e*) Find the area of the hexagon by using the area of GBC as a basis.

(*f*) Find the area of the hexagon by using the area of ABCG as a basis. } Ans. 93.5307 sq. in.

(*g*) Find the area of the hexagon by using the area of ABCD as a basis.

(*h*) Find the area of the circumscribing circle. Ans. 113.0976 sq. in.

(*i*) Find the area of the 6 segments.

Ans. 19.5669 sq. in.

30. A circle whose radius is 6 inches is circumscribed by a square, and also has a square inscribed (Art. 48, Fig. 2):

(*a*) Compare the length of the side of the circumscribed square, the diameter of the circle, and the diagonal of the inscribed square.

(*b*) Find the perimeter of the circumscribed square.

Ans. 48 in.

(*c*) Find the circumference of the circle.

Ans. 37.6992 in.

(*d*) Find the perimeter of the inscribed square.

Ans. 33.9408 in.

(*e*) Find the area of the circumscribed square.

Ans. 144 sq. in.

(*f*) Find the area of the circle.

Ans. 113.0976 sq. in.

(*g*) Find the area of the inscribed square.

Ans. 71.9986+ sq. in.

31. In 1899 the Williamsport Gas Company erected a cylindrical vat 100 feet in diameter and 100 feet high; required its capacity. Ans. 785,400 cu. ft.

SUPPLEMENT

DERIVATION OF THE PRISMOIDAL FORMULA FROM THE PRINCIPLES OF GEOMETRY

A Prismoid is composed of prisms, wedges, and pyramids. It is desired to find a common expression for finding the volume of these three volumes. From the principles of Geometry we get the following:

Volume of Prism = area of base × by altitude.
Volume of Wedge = area of base × by $\frac{1}{2}$ altitude.
Volume of Pyramid = area of base × by $\frac{1}{3}$ altitude.

Let B = { area of either base of prism. / area of the base of wedge. / area of the base of pyramid. }
and let a = length or altitude of each.

Then, Vol. Prism = Ba
Vol. Wedge = $\frac{Ba}{2}$
Vol. Pyramid = $\frac{Ba}{3}$

Reducing these three expressions to a common denominator, we have the following:

Vol. Prism = Ba = $\frac{6Ba}{6}$
Vol. Wedge = $\frac{Ba}{2}$ = $\frac{3Ba}{6}$
Vol. Pyramid = $\frac{Ba}{3}$ = $\frac{2Ba}{6}$

Now, let b = { the area of a cross-section of the pyramid at the apex. / the area of a cross-section of the wedge at its cutting edge. / the area of a cross-section of the prism at corresponding end of prism. }

and let m = { the area of a cross-section of the pyramid half way between base and apex. / the area of a cross-section of the wedge half way between base and sharp edge. / the area of a cross-section of the prism half way between the two bases. }

In the Prism { b = B, m = B }

In the Wedge { b = 0, m = $\frac{1}{2}$B }

In the Pyramid { b = 0, m = $\frac{1}{4}$B }

Substituting these values in the fractions having a common denominator, we have the following:

Vol. Prism = $\frac{6Ba}{6}$ = ($\frac{Ba}{6}$ + $\frac{Ba}{6}$ + $\frac{4Ba}{6}$) = $\frac{a}{6}$(B + B + 4B) = $\frac{a}{6}$(B + b + 4m).
Vol. Wedge = $\frac{3Ba}{6}$ = ($\frac{Ba}{6}$ + $\frac{0}{6}$ + $\frac{2Ba}{6}$) = $\frac{a}{6}$(B + 0 + 2B) = $\frac{a}{6}$(B + b + 4m).
Vol. Pyramid = $\frac{2Ba}{6}$ = ($\frac{Ba}{6}$ + $\frac{0}{6}$ + $\frac{Ba}{6}$) = $\frac{a}{6}$(B + 0 + B) = $\frac{a}{6}$(B + b + 4m).

Hence, the volume of the Prism, or Wedge, or Pyramid = $\frac{a}{6}$(B + b + 4m), and the volume of a Prismoid, therefore = $\frac{a}{6}$(B + b + 4m).

The rule may be expressed in words as follows: *Multiply the sum of both end areas and four times the area of a section half way between them, by one-sixth the altitude.*

A cylinder may be divided into an infinite number of prisms; a sphere or a cone into an infinite number of pyramids; frustums of pyramids and cones into prisms and pyramids, etc. It is evident, therefore, that this rule is very wide in its application.

If, for the purpose of association, we consider the capital letter B as the initial letter of the term "Base" (large), and the small letter b as the initial letter of "base" (small), and m the initial letter of the term "middle section," the rule may be quite easily remembered. In some solids, as the pyramid, cone, and their frustums, these letters will have special significance, since one base is larger than the other. This is not true, however, of all solids.

The following abbreviated process of using the formula may be used to advantage after the pupil has become thoroughly grounded in the process used in the chapter on Illustrative Problems.

PROBLEM.—What are the solid contents of the frustum of a square pyramid whose upper base is 6 feet square, lower base 10 feet square, and altitude 30 feet?

OPERATION.—

$$\begin{aligned} B &= 10 \times 10 = 100 \text{ sq. ft.} \\ b &= 6 \times 6 = 36 \text{ "} \\ 4m &= 4\ (8 \times 8) = 256 \text{ "} \\ & \overline{392} \times \tfrac{30}{6} = 1{,}960 \text{ cu. ft., Ans.} \end{aligned}$$

PROBLEM.—What is the volume of a sphere whose radius is 10 feet?

OPERATION.—

$$\begin{aligned} B &= 0.0 \\ b &= 0.0 \\ 4m &= 4\pi r^2 = 4(3.1416 \times 100) = 1{,}256.64 \\ & 1{,}256.64 \times \tfrac{20}{6} = 4{,}188.8 \text{ cu. ft., Ans.} \end{aligned}$$

MEASURES OF EXTENSION

LINEAR MEASURE

TABLE

12	inches (in.)	= 1 foot (ft.)
3	feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards / $16\frac{1}{2}$ feet		= 1 rod (rd.)
320	rods	= 1 mile (mi.)

1 mi. = 320 rds. = 1,760 yds. = 5,280 ft. = 63,360 in.

SCALE. — 320, $5\frac{1}{2}$, 3, 12.

OTHER DENOMINATIONS

3 barleycorns	= 1 inch (Used by shoemakers)
4 inches	= 1 hand (Used to measure the height of horses)
9 inches	= 1 span
8 spans	= 1 fathom (Used by sailors)
6 feet	= 1 fathom (Used to measure the depths at sea)
18 inches	= 1 common cubit
21.888 inches	= 1 sacred cubit
120 fathoms	= 1 cable's length
3 feet	= 1 pace
8 furlongs	= 1 mile
$1.152\frac{2}{3}$ common miles	= 1 geographical, or nautical mile = 1 knot
3 geographical miles	= 1 league
60 geographical, or 69.16 statute miles	= 1 degree { of latitude on a meridian, or of longitude on the equator
360 degrees	= the circumference of the earth

SURVEYORS' LINEAR MEASURE

TABLE

7.92 inches	= 1	link (l.)
25 links	= 1	rod (rd.)
4 rods	= 1	chain (ch.)
80 chains	= 1	mile (mi.)

1 mi. = 80 ch. = 320 rds. = 8,000 l. = 63,360 in.

A Gunter's Chain is the unit of measure, and is 4 rds. or 66 ft. long, and consists of 100 links.

SQUARE MEASURE

TABLE

144	square inches (sq. in.)	= 1	square foot (sq. ft.)
9	square feet	= 1	square yard (sq. yd.)
$30\frac{1}{4}$	square yards	= 1	square rod or perch [(sq. rd., P.)
160	square rods or perches	= 1	acre (A.)
640	acres	= 1	square mile (sq. mi.)

1 sq. mi. = 640 A. = 102,400 sq. rds. = 3,097,600 sq. yds. = 27,878,400 sq. ft. = 4,014,489,600 sq. in.

SCALE. — 640, 160, $30\frac{1}{4}$, 9, 144.

SURVEYORS' SQUARE MEASURE

TABLE

625	square links (sq. l.)	= 1 pole (P.)
16	poles	= 1 square chain (sq. ch.)
10	square chains	= 1 acre (A.)
640	acres	= 1 square mile, or section (sq. mi., sec.)
36	square miles (6 miles square)	= 1 township (Tp.)

1 Tp. = 36 sq. mi. = 23,040 A. = 230,400 sq. ch. = 3,686,400 P. = 2,304,000,000 sq. l.

SCALE. — 36, 640, 10, 16, 625.

CUBIC MEASURE

TABLE

1,728	cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27	cubic feet	= 1 cubic yard (cu. yd.)
16	cubic feet	= 1 cord foot (cd. ft.)
8 cord feet, or 128	cubic feet	= 1 cord of wood (cd.)
$24\frac{3}{4}$	cubic feet	= 1 perch of stone, or of masonry (Pch.)

1 cu. yd. = 27 cu. ft. = 46,656 cu. in.

SCALE.— 27, 1,728.

CIRCULAR OR ANGULAR MEASURE

TABLE

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
30 degrees	= 1 sign (S.)
12 signs, or 360 degrees	= 1 circle (cir.)

1 cir. = 12 S. = 30° = 21,600′ = 1,296,000″.

SCALE.— 12, 30, 60, 60.

DUODECIMALS

TABLE

12 fourths (″″)	= 1 third (‴)
12 thirds	= 1 second (″)
12 seconds	= 1 prime (′)
12 primes, or inches	= 1 foot (ft.)

1 ft. = 12′ = 144″ = 1,728‴ = 20,736⁗.

SCALE.— 12, 12, 12, 12.

MISCELLANEOUS

2,150.42 cubic inches = 1 Winchester bushel (bu.)

COMPARATIVE TABLE OF WEIGHTS

	Avoirdupois	*Troy*	*Apothecaries'*
1 pound =	7,000 grains =	5,760 grains =	5,760 grains
1 ounce =	437.5 grains =	480 grains =	480 grains

COMPARATIVE TABLE OF MEASURES OF CAPACITY

	Cu. in. in one gallon	*Cu. in. in one quart*	*Cu. in. in one pint*	*Cu. in. in one gill*
Liquid Measure	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry Measure (half-peck) .	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

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